Chapter 12 Microwave Amplifier Design

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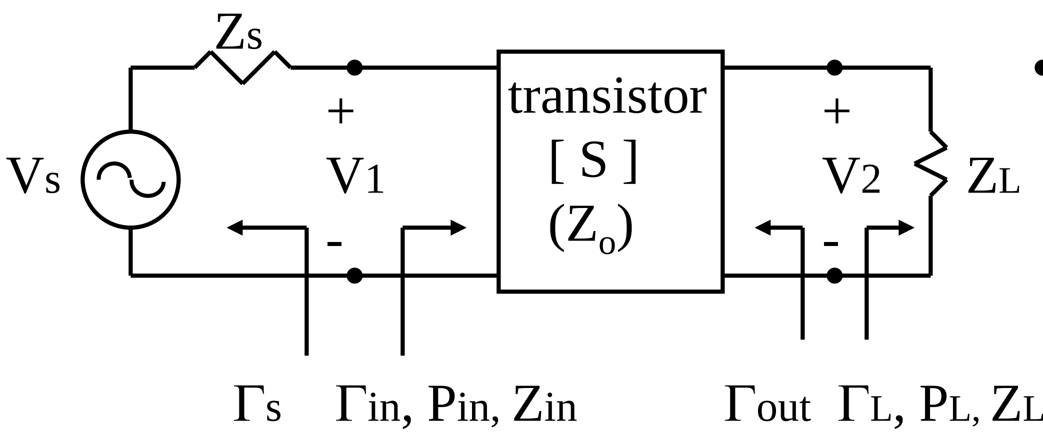
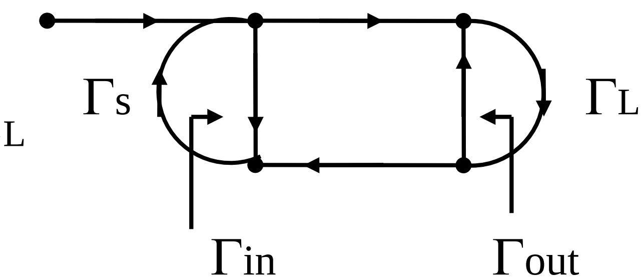
12.1 Two-port power gains





power gain $G \equiv \frac{P_L}{P_{in}}(S, \Gamma_L)$

available power gain $G_A \equiv \frac{P_{avn}}{P_{avs}}(S, \Gamma_S)$

transducer power gain $G_T \equiv \frac{P_L}{P_{avs}}(S, \Gamma_S, \Gamma_L)$

$P_{in}(\Gamma_{in}), P_{avs}(\Gamma_s) = P_{in}\big|_{\Gamma_{in}=\Gamma_S^*}, P_L(\Gamma_L), P_{avn}(\Gamma_{out}) = P_L\big|_{\Gamma_L=\Gamma_{out}^*}$

Discussion

1.

$$V_1 = V_s \frac{Z_{in}}{Z_s + Z_{in}} = V_1^+ + V_1^- = V_1^+(1+\Gamma_{in}),$$

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o}, \Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o}$$

$$\rightarrow V_1^+ = \frac{V_s}{1+\Gamma_{in}} \frac{Z_{in}}{Z_s + Z_{in}} = \frac{V_s}{2} \frac{1-\Gamma_s}{1-\Gamma_s\Gamma_{in}}$$

$$P_{in} = P_s(1-|\Gamma_{in}|^2) = \frac{1}{2}\frac{|V_1^+|^2}{Z_o}(1-|\Gamma_{in}|^2) = \frac{|V_s|^2}{8Z_o}\frac{|1-\Gamma_s|^2}{|1-\Gamma_s\Gamma_{in}|^2}(1-|\Gamma_{in}|^2)$$

2. $V_2^- = S_{21}V_1^+ + S_{22}V_2^+, V_2^+ = \Gamma_L V_2^-, V_1^+ = \frac{V_s}{2}\frac{1-\Gamma_s}{1-\Gamma_s\Gamma_{in}}$

$$\rightarrow V_2^- = \frac{V_s}{2}\frac{S_{21}(1-\Gamma_s)}{(1-S_{22}\Gamma_L)(1-\Gamma_s\Gamma_{in})}$$

$$P_L = P_{out}(1-|\Gamma_L|^2) = \frac{1}{2}\frac{|V_2^-|^2}{Z_o}(1-|\Gamma_L|^2) = \frac{|V_s|^2}{8Z_o}\frac{|S_{21}|^2|1-\Gamma_s|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_s\Gamma_{in}|^2}(1-|\Gamma_L|^2)$$

3. $$P_{avs} = P_{in}\big|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_s|^2}{8Z_o}\frac{|1-\Gamma_s|^2}{1-|\Gamma_s|^2}$$

$$P_{avn} = P_L\big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_s|^2}{8Z_o}\frac{|S_{21}|^2|1-\Gamma_s|^2(1-|\Gamma_{out}|^2)}{|1-S_{22}\Gamma_{out}^*|^2|1-\Gamma_s\Gamma_{in}|^2}, \Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

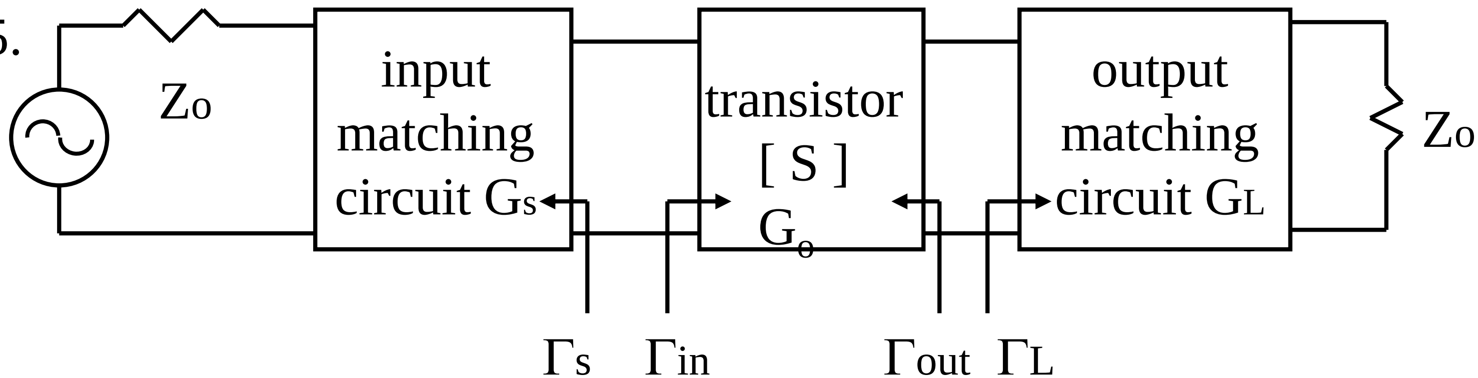
$$\rightarrow P_{avn} = \frac{|V_s|^2}{8Z_o}\frac{|S_{21}|^2|1-\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)}$$

4. $$G(S,\Gamma_L) \equiv \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2}$$

$$G_A(S,\Gamma_s) \equiv \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{(1-|\Gamma_{out}|^2)|1-S_{11}\Gamma_s|^2}$$

$$G_T(S,\Gamma_s,\Gamma_L) \equiv \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_s\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2}(=|S_{21}|^2,\ \text{if } \Gamma_s = \Gamma_L = 0)$$

5.



$$G_T = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s\Gamma_{in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = G_s G_o G_L$$

$$\Gamma_{in} = \Gamma_s^*, \Gamma_{out} = \Gamma_L^* \rightarrow G_{T\max} = \frac{1}{1-|\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$S_{12} = 0,\ \text{unilateral transducer gain } G_{TU} = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$\Gamma_s = S_{11}^*, \Gamma_L = S_{22}^* \rightarrow G_{TU\max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} \Rightarrow \textit{selection of transistor}$$

6. Ex.12.1 A Si BJT@1GHz

$$S_{11}=0.38\angle-158°, S_{12}=0.11\angle 54°, S_{21}=3.5\angle 80°, S_{22}=0.4\angle-43°$$

$Z_S=25\Omega$, $Z_L=40\Omega$, $Z_o=50\Omega$

$$\Gamma_s=\frac{Z_s-Z_o}{Z_s+Z_o}=-0.333, \Gamma_L=\frac{Z_L-Z_o}{Z_L+Z_o}=-0.111$$

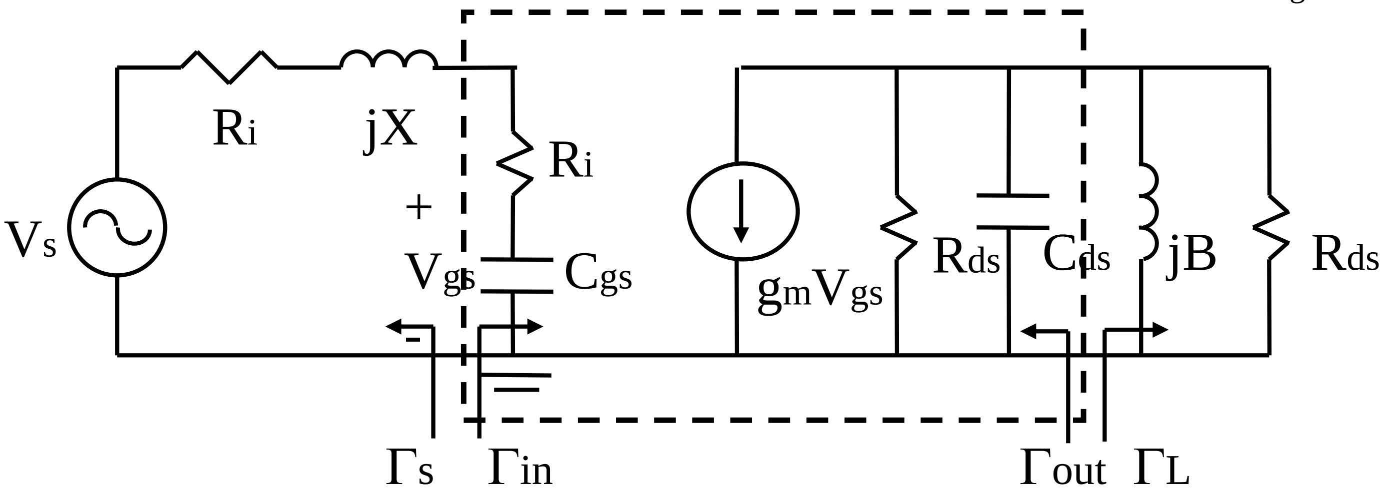
$$\Gamma_{in}=S_{11}+\frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}=0.365\angle-152°$$

$$\Gamma_{out}=S_{22}+\frac{S_{12}S_{21}\Gamma_s}{1-S_{11}\Gamma_s}=0.545\angle-43°$$

$$G=13.1, G_A=19.8, G_T=12.6$$

$$G_T=\frac{P_L}{P_{avs}}<\frac{P_L}{P_{in}}=G, G_A=\frac{P_{avn}}{P_{avs}}<\frac{P_L}{P_{avs}}=G_T$$

7. conjugate match using FET equivalent circuit (S_{12}=0, or C_{gd}=0)



$$Z_{in} = Z_S^* \rightarrow \frac{1}{wC_{gs}} = X, Z_{out} = Z_L^* \rightarrow wC_{ds} = -B$$

$$V_{gs} = \frac{V_s}{2R_i}\frac{1}{jwC_{gs}}$$

$$G_{TU} = \frac{P_L}{P_{avs}} = \frac{\frac{1}{2}(\frac{1}{2}\left|g_m V_{gs}\right|)^2 R_{ds}}{\frac{1}{2}(\frac{1}{2}V_s)^2 / R_i} = \frac{g_m^2 R_{ds}}{4w^2 R_i C_{gs}^2} = \frac{R_{ds}}{4R_i}(\frac{f_T}{f})^2 : 6dB/octave,\ f_T = \frac{g_m}{2\pi C_{gs}}$$

12.2 Stability (S, f)

$$\text{unconditional stable } \forall\, Z_s, Z_L \;\rightarrow\; |\Gamma_{\text{in}}| < 1, |\Gamma_{\text{out}}| < 1$$

$$\text{conditional stable } \exists\, Z_s, Z_L \;\rightarrow\; |\Gamma_{\text{in}}| < 1, |\Gamma_{\text{out}}| < 1$$

Discussion

1. $S_{12} = 0, \quad |\Gamma_{\text{in}}| < 1, |\Gamma_{\text{out}}| < 1 \quad \rightarrow \quad |S_{11}| < 1, |S_{22}| < 1$

2. $|\Gamma_{in}| = \left| S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1 \rightarrow$ output stability circle $|\Gamma_{\text{L}} - C_L| = R_L$

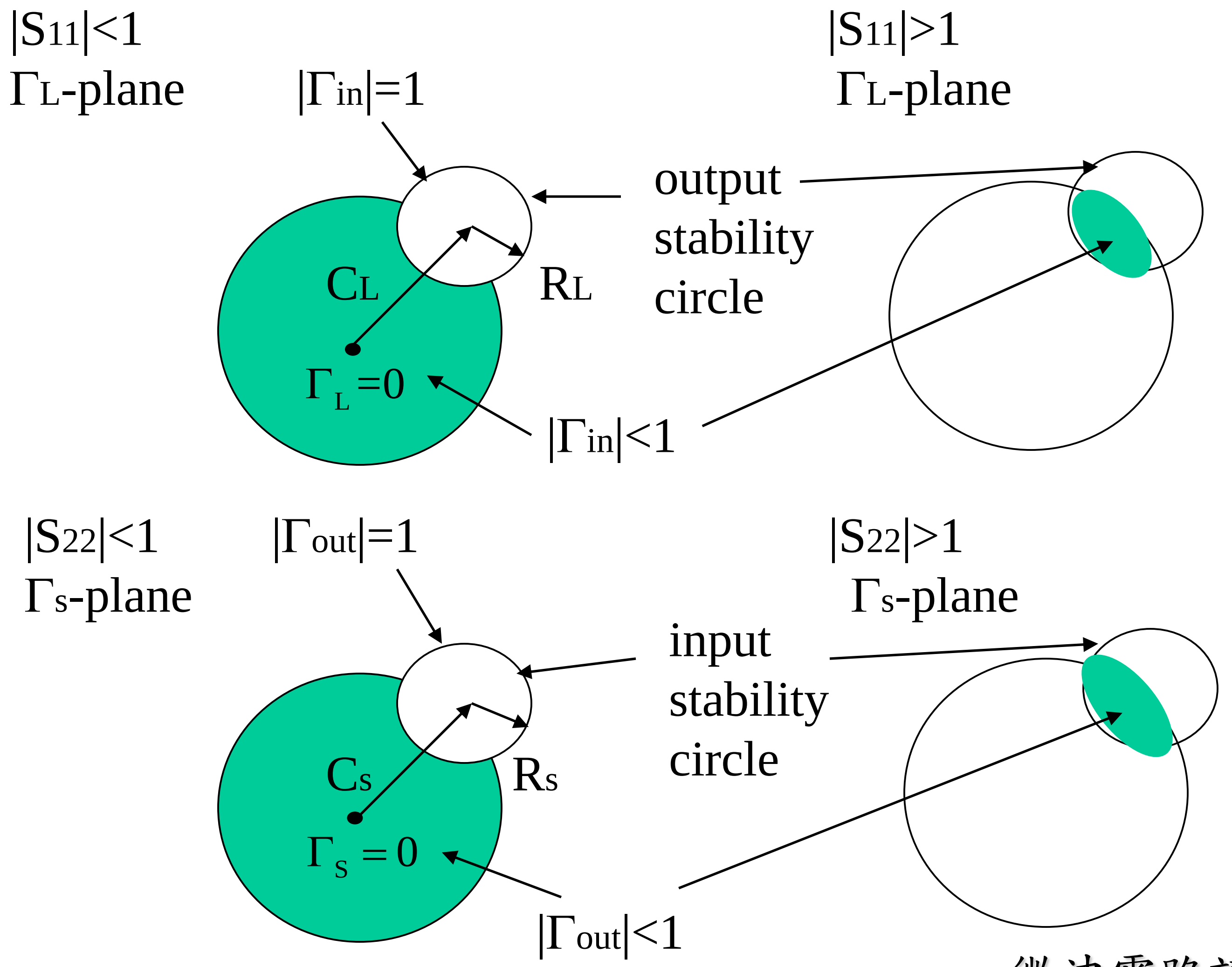
$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2},\; R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

$|\Gamma_{out}| = \left| S_{22} + \dfrac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| = 1 \rightarrow$ input stability circle $|\Gamma_{\text{s}} - C_s| = R_s$

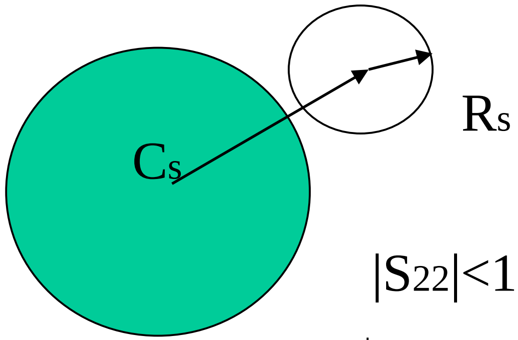
$$C_s = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2},\; R_s = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

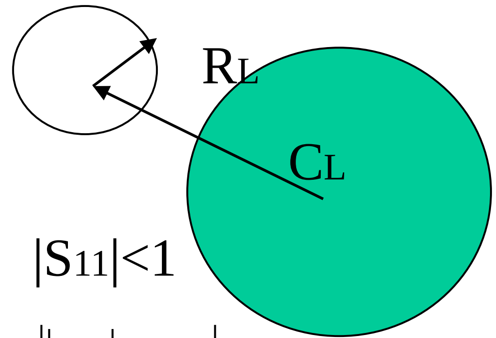
(derivation in p.565)

3. conditional stable



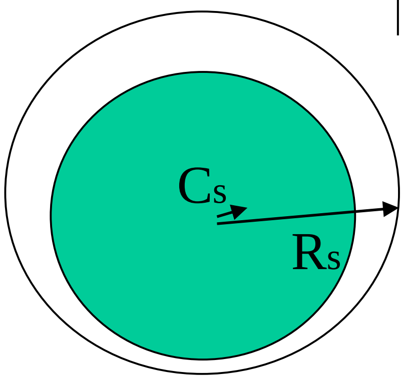
4. unconditional stable, stability factor K

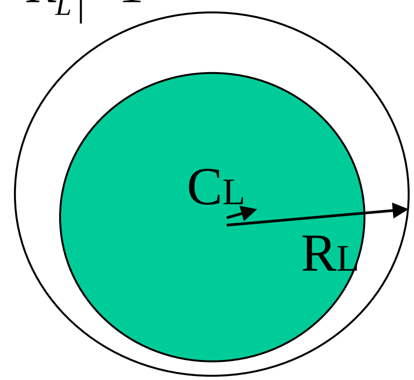




$|S_{22}|<1$ $\quad$ $|S_{11}|<1$

$\left||C_S|-R_S\right|>1$ $\quad$ $\left||C_L|-R_L\right|>1$





unconditional stable $|S_{11}|<1, |S_{22}|<1, \quad \left||C_s|-R_s\right|>1, \left||C_L|-R_L\right|>1$

$$\leftrightarrow |\Delta|<1, K=\frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{12}S_{21}|}>1, \Delta=\det[S]$$:Rollet's condition

(derivation in p.568 and 569)

$\Rightarrow K,$ *selection of transistor*

5. In practice, one should consider stability over a wide bandwidth for the possible oscillations.

6. Ex.12.2 Triquint T1G6000528 @ 1.9GHz, Z_o=50Ω

$$S_{11} = 0.869\angle -159°, S_{12} = 0.031\angle -9°, S_{21} = 4.25\angle 61°,$$

$$S_{22} = 0.507\angle -117°$$

$$|\Delta| = 0.336 < 1, K = 0.383 < 1$$

input stability circle $C_s = 1.09\angle 162°, R_s = 0.205$

output stability circle $C_L = 1.59\angle 132°, R_L = 0.915$

(p.570, Fig.12.6)

12.3 Single-stage transistor amplifier design

- conjugate match (maximum transducer power gain)

if $|\Delta| < 1,\ K > 1$

$\rightarrow$ input and output simultaneously conjugate match $\Gamma_{in} = \Gamma_s^*, \Gamma_{out} = \Gamma_L^*$

$$\rightarrow G_T = G_{T\max} = \frac{1}{1-|\Gamma_s|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \left|\frac{S_{21}}{S_{12}}\right|(K-\sqrt{K^2-1})$$

$$\Gamma_s^* = \Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} \rightarrow \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$\Gamma_L^* = \Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1-S_{11}\Gamma_s} \rightarrow \Gamma_s = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2, B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^*, C_2 = S_{22} - \Delta S_{11}^*$$

(derivation in p. 571 and 572)

Discussion

1. linear amplifier design procedure

if $|\Delta|<1$, $K>1$ then uses input and output simultaneously conjugate matches for G_{Tmax}

if $K<1$ then draws input and output stability circles to see if input and output simultaneously conjugate matches possible, otherwise selects proper Γ_s and Γ_L for gain or noise figure considerations.

2.

$$S_{12}=0\rightarrow\Gamma_s=S_{11}^*,\Gamma_L=S_{22}^*$$

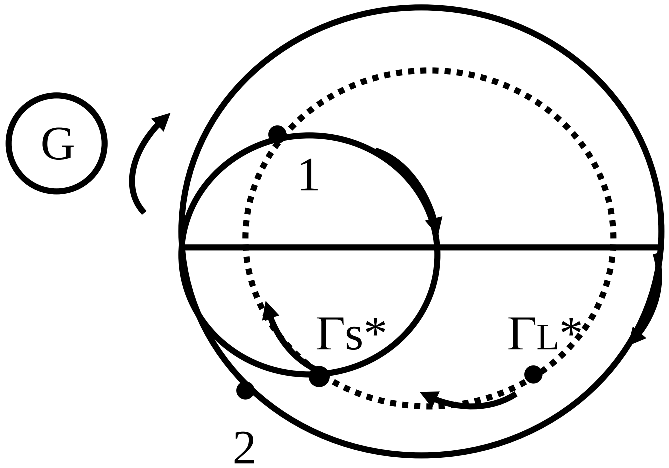
$$G_{TU\,max}=\frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}=G_{s\,max}|S_{21}|^2G_{L\,max}$$

3. Ex.12.3 FET @ 4GHz

$$S_{11}=0.72\angle-116°,S_{12}=0.03\angle57°,S_{21}=2.6\angle76°,S_{22}=0.73\angle-54°$$

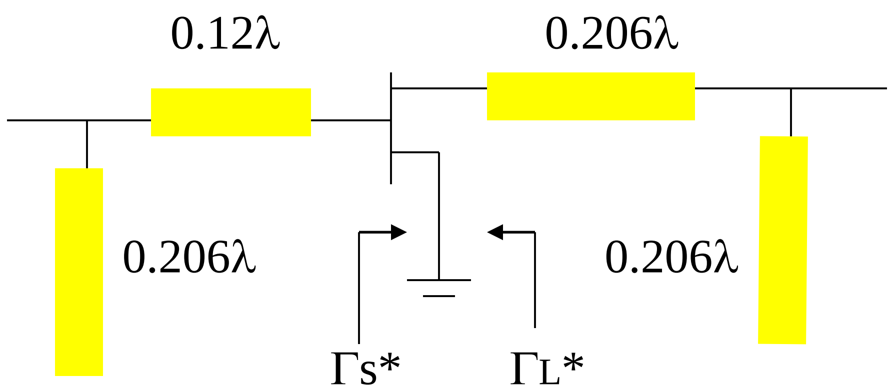
$$|\Delta|=0.488<1,K=1.195>1\rightarrow\Gamma_s=0.872\angle123°,\Gamma_L=0.876\angle61°$$

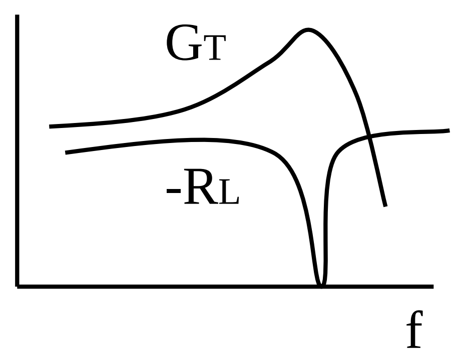
$$G_{T\max}=6.2+8.3+2.22=16.7dB$$



$$\Gamma_s^* = 0.872\angle -123^\circ$$
$$\Gamma_L^* = 0.876\angle -61^\circ$$

1. y=1-j3.5
2. y=j3.5





frequency response (p.575, Fig.12.7)

• constant gain circle (S_{12}=0, unilateral assumption)

$$G_T = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s\Gamma_{in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}, S_{12}=0 \to \Gamma_{in}=S_{11}$$

$$G_s = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}, G_{s\max} = \frac{1}{1-|S_{11}|^2}, G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}, G_{L\max} = \frac{1}{1-|S_{22}|^2}$$

$$g_s \equiv \frac{G_s}{G_{s\max}} \to \text{constant gain circle in } \Gamma_{\mathrm{S}}\text{-plane } |\Gamma_{\mathrm{S}} - C_S| = R_S$$

$$g_L \equiv \frac{G_L}{G_{L\max}} \to \text{constant gain circle in } \Gamma_{\mathrm{L}}\text{-plane } |\Gamma_{\mathrm{L}} - C_L| = R_L$$

$$C_s = \frac{g_s S_{11}^*}{1-(1-g_s)|S_{11}|^2}, R_s = \frac{\sqrt{1-g_s}(1-|S_{11}|^2)}{1-(1-g_s)|S_{11}|^2}$$

$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2}, R_L = \frac{\sqrt{1-g_L}(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}$$

(derivation in p. 576 and 577)

Discussion

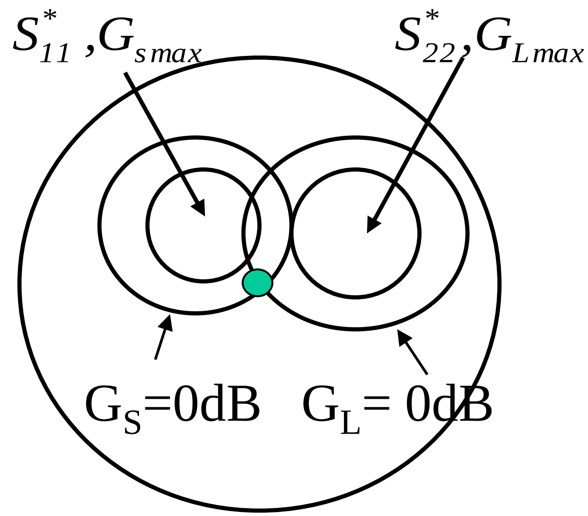
1. Gs=0dB and G_L= 0dB circles pass through the Smith chart center.

$$G_s=\frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}(=0dB)=1, G_{s\max}=\frac{1}{1-|S_{11}|^2},$$

$$G_L=\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}(=0dB)=1, G_{L\max}=\frac{1}{1-|S_{22}|^2}$$

$$g_s=\frac{G_s}{G_{s\max}}=\frac{1}{G_{s\max}}=1-|S_{11}|^2\rightarrow 1-g_s=|S_{11}|^2,$$

$$g_L=\frac{G_L}{G_{L\max}}=\frac{1}{G_{L\max}}=1-|S_{22}|^2\rightarrow 1-g_L=|S_{22}|^2$$



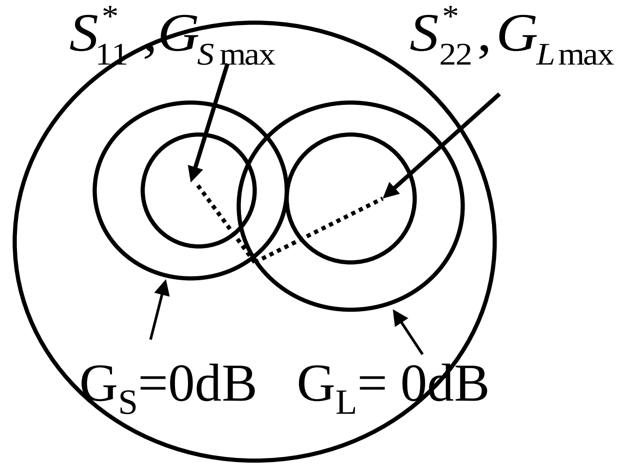
constant gain circles $|\Gamma_S-C_s|=R_s$, $|\Gamma_L-C_L|=R_L$

$$C_s=\frac{g_sS_{11}^*}{1-(1-g_s)|S_{11}|^2}=\frac{(1-|S_{11}|^2)S_{11}^*}{1-|S_{11}|^4}=\frac{S_{11}^*}{1+|S_{11}|^2}, R_s=\frac{\sqrt{1-g_s}(1-|S_{11}|^2)}{1-(1-g_s)|S_{11}|^2}=\frac{|S_{11}|(1-|S_{11}|^2)}{1-|S_{11}|^4}=\frac{|S_{11}|}{1+|S_{11}|^2}$$

$$C_L=\frac{g_LS_{22}^*}{1-(1-g_L)|S_{22}|^2}=\frac{(1-|S_{22}|^2)S_{22}^*}{1-|S_{22}|^4}=\frac{S_{22}^*}{1+|S_{22}|^2}, R_L=\frac{\sqrt{1-g_L}(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}=\frac{|S_{22}|(1-|S_{22}|^2)}{1-|S_{22}|^4}=\frac{|S_{22}|}{1+|S_{22}|^2}$$

$$\rightarrow |C_s|=R_s, |C_L|=R_L\rightarrow \Gamma_S=\Gamma_L=0$$

2. Centers of constant gain circles are distributed along the lines from S_{11}* and S_{22}* to the Smith chart center, respectively.



$$C_s = \frac{g_s S_{11}^*}{1-(1-g_s)|S_{11}|^2}, C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2}$$

$$g_s = 1, g_L = 1 \rightarrow C_{s,1} = S_{11}^*, C_{L,1} = S_{22}^*$$

$$G_s = 1, G_L = 1 \rightarrow C_{s,2} = \frac{S_{11}^*}{1+|S_{11}|^2}, C_{L,2} = \frac{S_{22}^*}{1+|S_{22}|^2}$$

$$\Rightarrow \tan^{-1}\frac{\mathrm{Re}(C_{s,1})}{\mathrm{Im}(C_{s,1})} = \tan^{-1}\frac{\mathrm{Re}(C_{s,2})}{\mathrm{Im}(C_{s,2})}, \tan^{-1}\frac{\mathrm{Re}(C_{L,1})}{\mathrm{Im}(C_{L,1})} = \tan^{-1}\frac{\mathrm{Re}(C_{L,2})}{\mathrm{Im}(C_{L,2})}$$

2. $$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$$

$$U = \frac{|S_{11}||S_{21}||S_{12}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$ unilaterial figure of merit

3. Ex.12.4 design an amplifier with G_T=11dB @ 4GHz

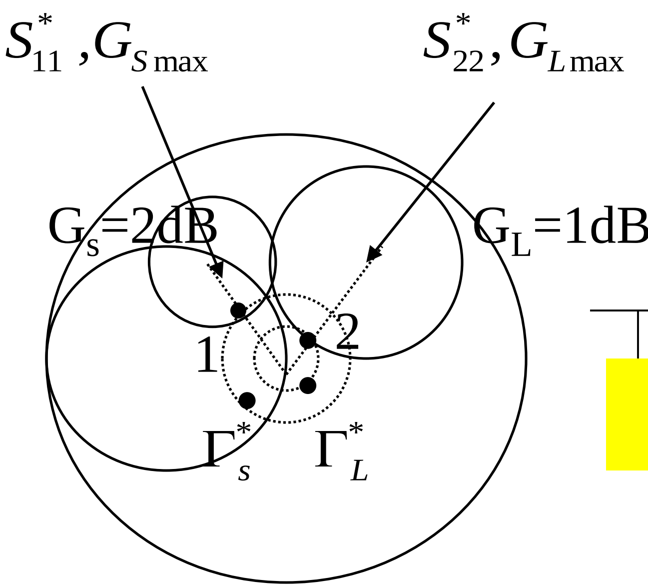
$$S_{11} = 0.75\angle -120°, S_{12} = 0, S_{21} = 2.5\angle 80°, S_{22} = 0.6\angle -70°$$

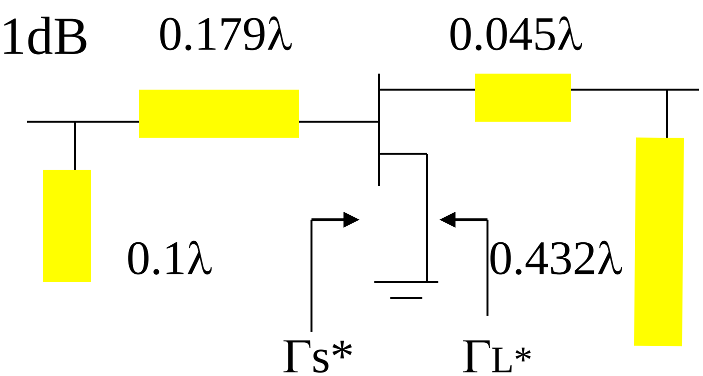
$$G_{TU\max} = 3.6 + 8 + 1.9 = 13.5dB$$

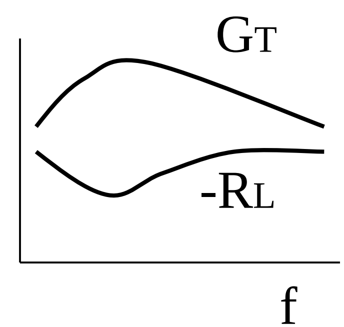
$$\text{choose } G_{TU} = 2 + 8 + 1 = 11dB$$

$$1.\ \Gamma_s = 0.33\angle 120°, \Gamma_s^* = 0.33\angle -120°$$

$$2.\ \Gamma_L = 0.22\angle 70°, \Gamma_L^* = 0.22\angle -70°$$







frequency response (p.579, Fig.12.8)

• constant noise figure circle

for a two-port amplifier

$$F = F_{\min} + \frac{R_N}{G_s}\left|Y_s - Y_{opt}\right|^2 = F_{\min} + \frac{4R_N}{Z_o}\frac{\left|\Gamma_s - \Gamma_{opt}\right|^2}{(1-\left|\Gamma_s\right|^2)\left|1+\Gamma_{opt}\right|^2}$$

noise parameter: $F_{\min}, Y_{opt}, R_N$ equivalent noise resistance of transistor

$$N \equiv \frac{\left|\Gamma_s - \Gamma_{opt}\right|^2}{1-\left|\Gamma_s\right|^2} = \frac{F - F_{\min}}{4R_N / Z_o}\left|1+\Gamma_{opt}\right|^2$$

$\rightarrow$ constant noise figure circle $\left|\Gamma_s - C_F\right| = R_F$

$$C_F = \frac{\Gamma_{opt}}{N+1}, R_F = \frac{\sqrt{N(N+1-\left|\Gamma_{opt}\right|^2)}}{N+1}$$

(derivation in p. 580 and 581)

<u>Discussion</u>

1. Ex.12.5 design a LNA with F=2dB and max. gain @ 4GHz

$$S_{11}=0.6\angle-60°, S_{12}=0.05\angle 26°, S_{21}=1.9\angle 81°, S_{22}=0.5\angle-60°$$

$$F_{\min}=1.6dB, \Gamma_{opt}=0.62\angle 100°, R_N=20\Omega$$

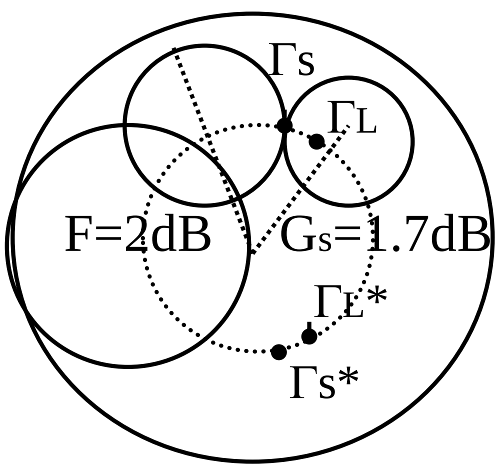
$$U=0.059$$

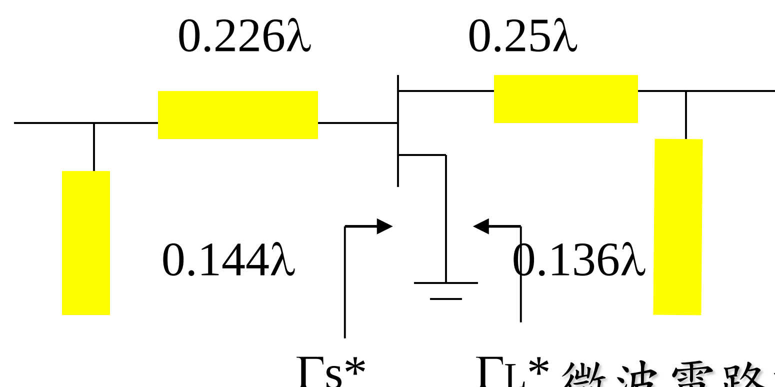
$$0.89=\frac{1}{(1+U)^2}<\frac{G_T}{G_{TU}}<\frac{1}{(1-U)^2}=1.13, -0.5dB<G_T-G_{TU}<0.53dB$$

$$F=2dB\rightarrow C_F=0.56\angle 100°, R_F=0.24$$

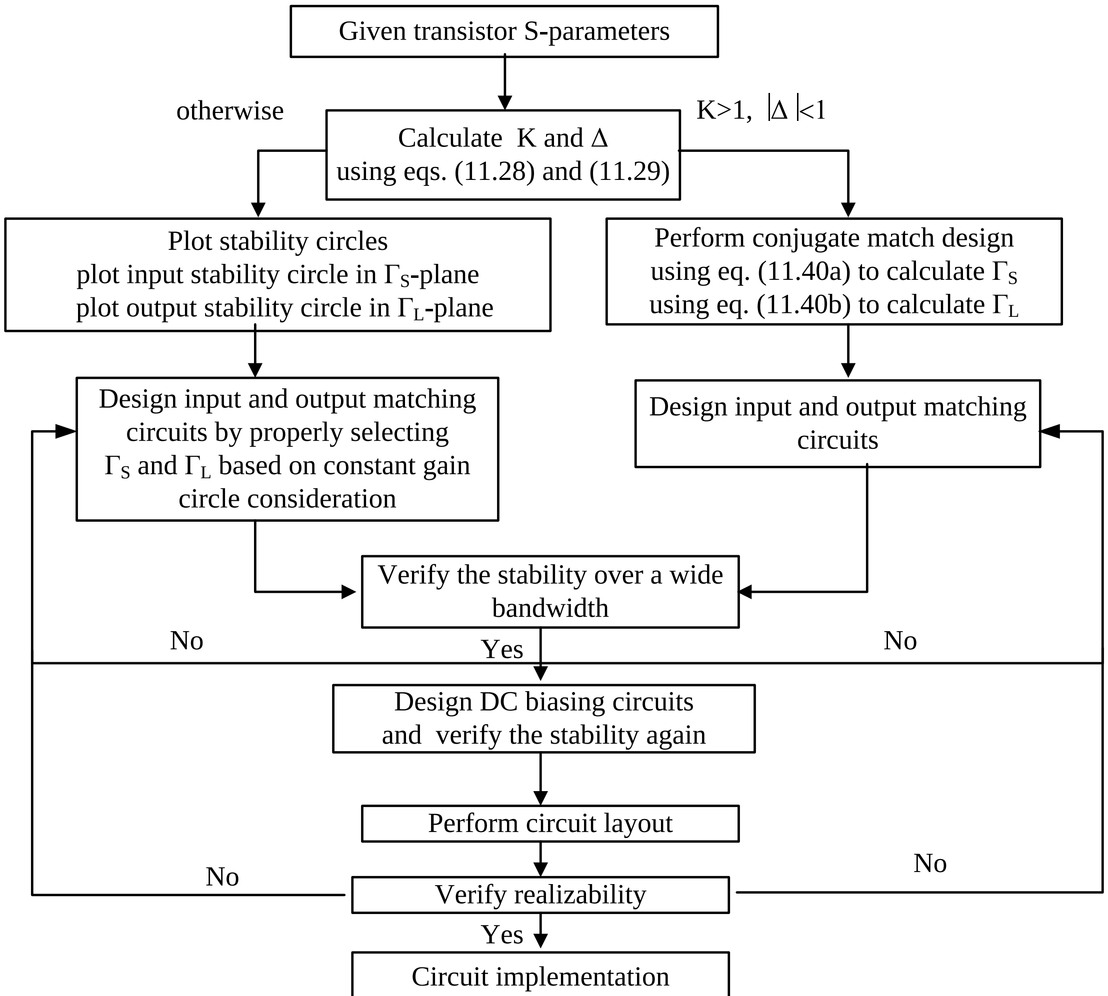
$$G_s=1.7dB\rightarrow C_s=0.58\angle 60°, R_s=0.15\rightarrow \Gamma_s=0.53\angle 75°$$

$$\Gamma_L=S_{22}^*=0.5\angle 60°\rightarrow G_L=\frac{1}{1-|S_{22}|^2}=1.25dB\rightarrow G_{TU}=1.7+|S_{21}|^2+1.25=8.53dB$$



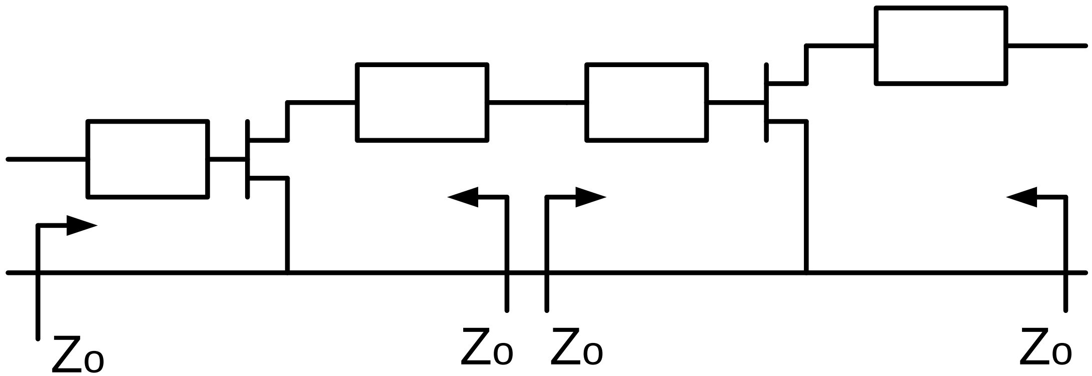


2. Approach for single-stage linear amplifier design

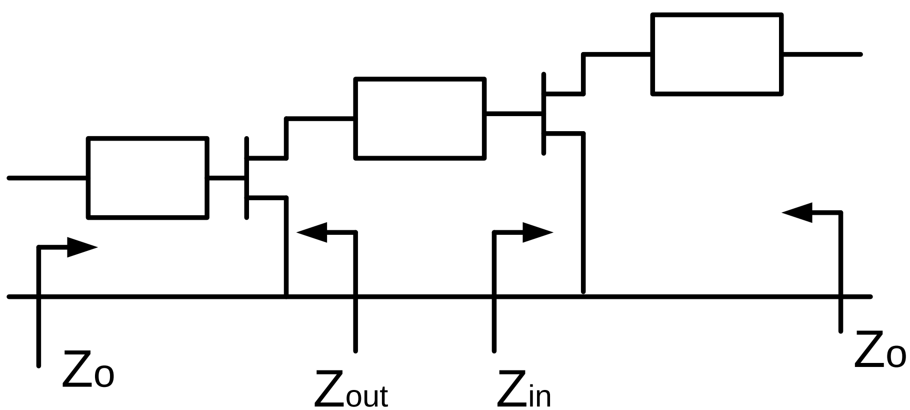


3. Two approaches for multi-stage amplifier design

(1)

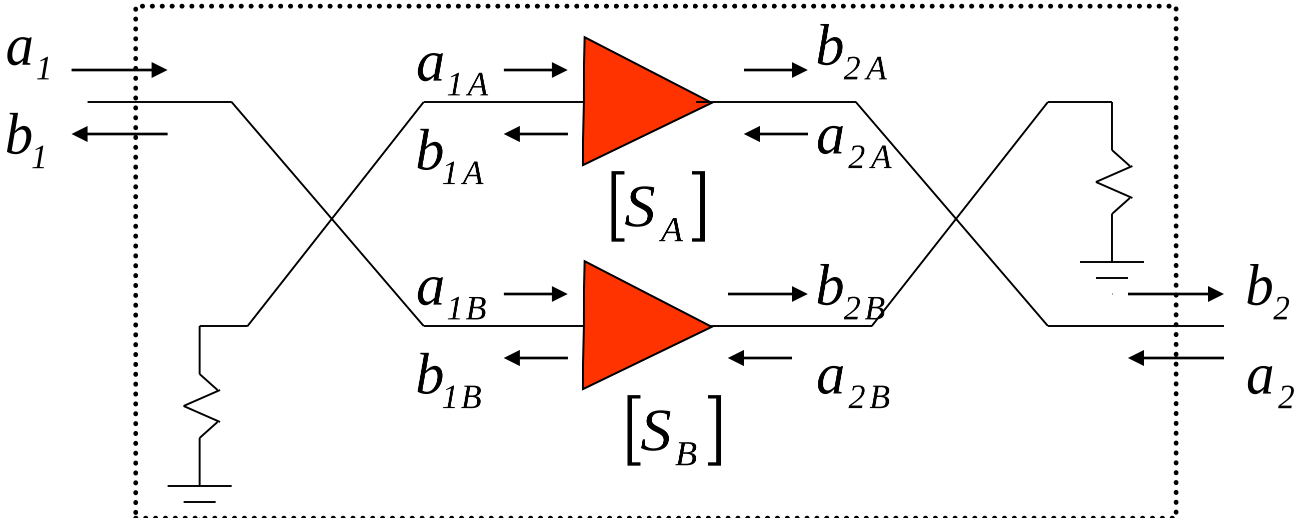


(2)



12.4 Broadband transistor amplifier design

- Balanced amplifier

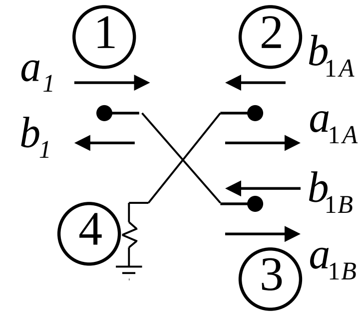


$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} S_{11A} - S_{11B} & -j(S_{12A} + S_{12B}) \\ -j(S_{21A} + S_{21B}) & -(S_{22A} - S_{22B}) \end{bmatrix}$$

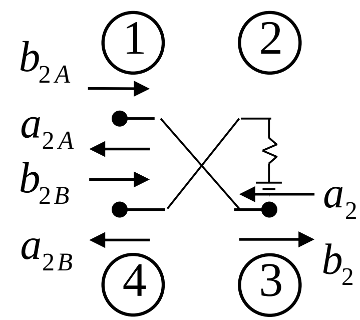
Discussion

1. Derivation of S-parameters

$$90^{\circ}\ \text{hybrid} \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{-j}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{-j}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}, \begin{bmatrix} b_1 \\ a_{1A} \\ a_{1B} \\ - \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{-j}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{-j}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ b_{1A} \\ b_{1B} \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} b_{1A,B} \\ b_{2A,B} \end{bmatrix} = \begin{bmatrix} S_{11A,B} & S_{12A,B} \\ S_{21A,B} & S_{22A,B} \end{bmatrix} \begin{bmatrix} a_{1A,B} \\ a_{2A,B} \end{bmatrix}, \begin{bmatrix} a_{2A} \\ - \\ b_2 \\ a_{2B} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{-j}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{-j}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} b_{2A} \\ 0 \\ a_2 \\ b_{2B} \end{bmatrix}$$



$$b_1 = \frac{1}{\sqrt{2}} b_{1A} + \frac{-j}{\sqrt{2}} b_{1B}$$

$$= \frac{1}{\sqrt{2}} (S_{11A} a_{1A} + S_{12A} a_{2A}) + \frac{-j}{\sqrt{2}} (S_{11B} a_{1B} + S_{12B} a_{2B})$$

$$= \frac{1}{\sqrt{2}} (S_{11A} \frac{1}{\sqrt{2}} a_1 + S_{12A} \frac{-j}{\sqrt{2}} a_2) + \frac{-j}{\sqrt{2}} (S_{11B} \frac{-j}{\sqrt{2}} a_1 + S_{12B} \frac{1}{\sqrt{2}} a_2)$$

$$= \frac{1}{2} (S_{11A} - S_{11B}) a_1 + \frac{-j}{2} (S_{12A} + S_{12B}) a_2$$

$$b_2 = \frac{-j}{\sqrt{2}} b_{2A} + \frac{1}{\sqrt{2}} b_{2B}$$

$$= \frac{-j}{\sqrt{2}} (S_{21A} a_{1A} + S_{22A} a_{2A}) + \frac{1}{\sqrt{2}} (S_{21B} a_{1B} + S_{22B} a_{2B})$$

$$= \frac{-j}{\sqrt{2}} (S_{21A} \frac{1}{\sqrt{2}} a_1 + S_{22A} \frac{-j}{\sqrt{2}} a_2) + \frac{1}{\sqrt{2}} (S_{21B} \frac{-j}{\sqrt{2}} a_1 + S_{22B} \frac{1}{\sqrt{2}} a_2)$$

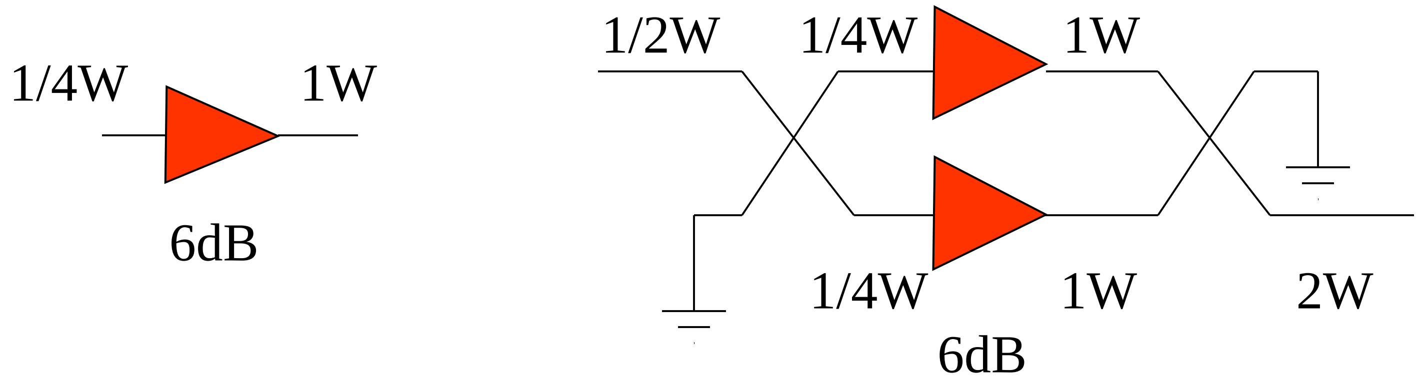
$$= \frac{-j}{2} (S_{21A} + S_{21B}) a_1 + \frac{-1}{2} (S_{22A} - S_{22B}) a_2$$

2. amplifier A=amplifier B, good i/p and o/p match
$\rightarrow$ good stability

$$\begin{bmatrix} 0 & -jS_{12A} \\ -jS_{21A} & 0 \end{bmatrix}$$

3. high reliablity and less tuning work
4. I/p and o/p matching are improved by two 90° hybrids, and mismatch reflections are absorbed by two resistors.
5. If one transistors fails, gain drops 6dB.
$\rightarrow$ graceful degradation
6. disadvantages: larger size and lower efficiency
7. Bandwidth is limited by two hybrids.

8. Power amplifier application

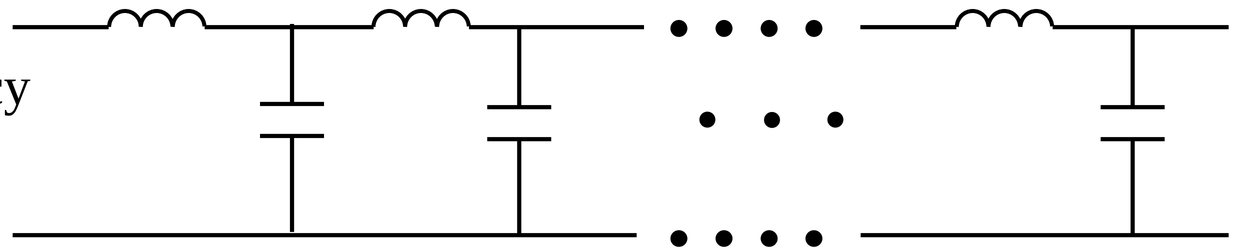


9. Balanced amplifiers can be implemented in a tree structure with a very high power in radar and communication applications.

10. Ex 12.7, two amplifiers of ex.12.4 are implemented as a balanced amplifier to improve its i/p and o/p return loss at 4 GHz. Then, the stub lengths are optimize to give better matching and gain flatness from 3 to 5 GHz bandwidth.
frequency response (p.588, Fig.12.11)

- Distributed (traveling wave) amplifier

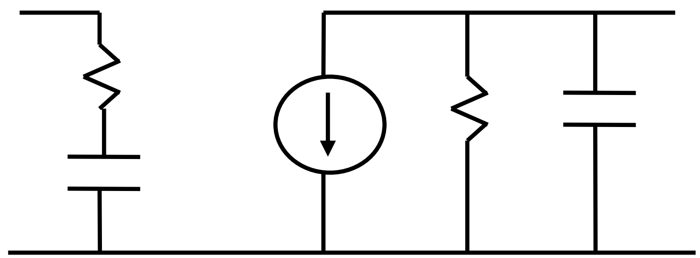
TEM line "extreme wide operation bandwidth"

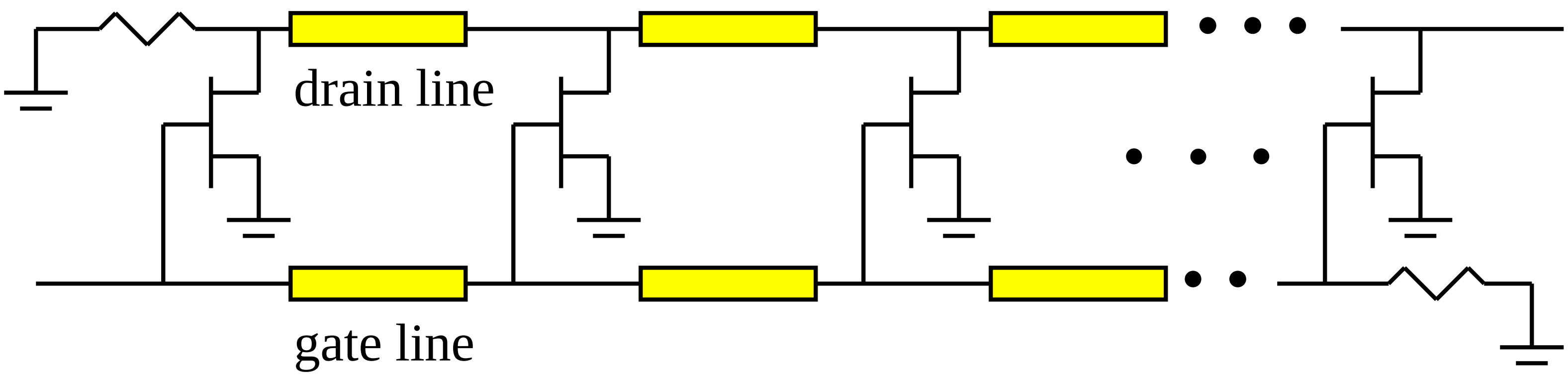


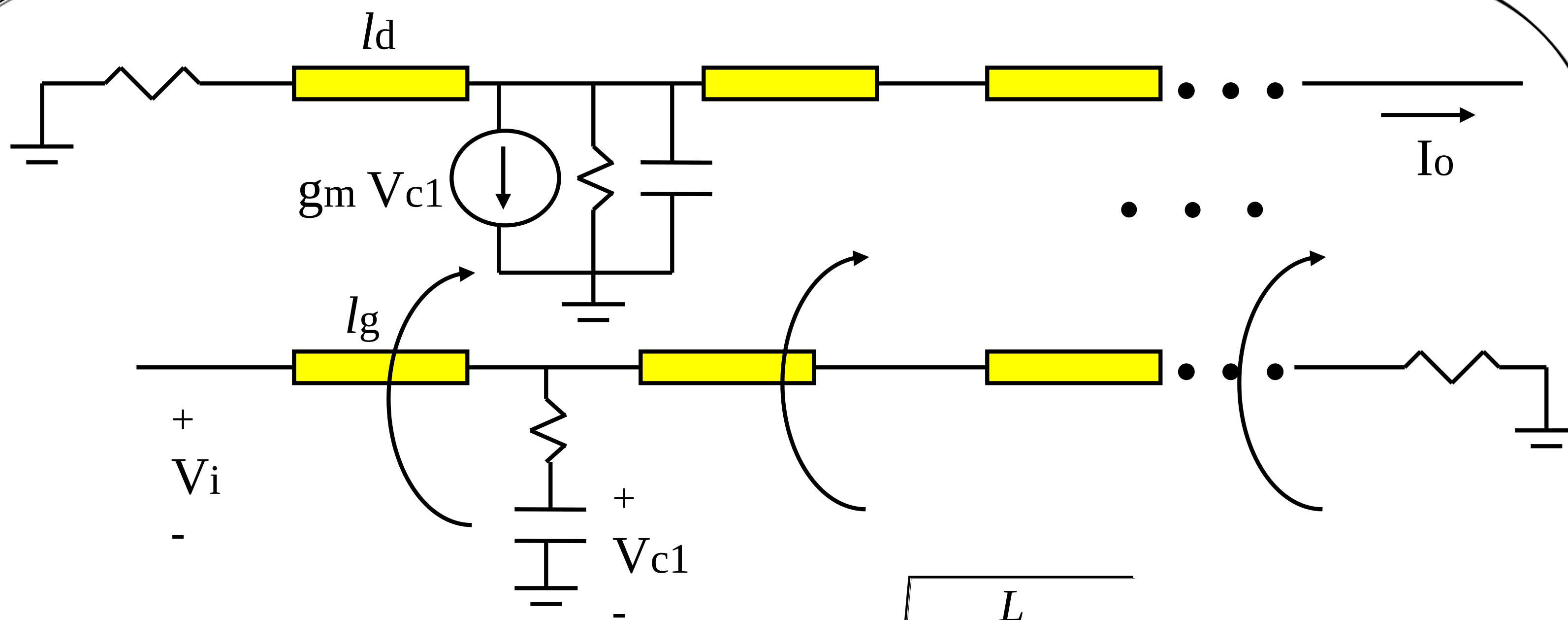
cut off frequency

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

FET equivalent circuit

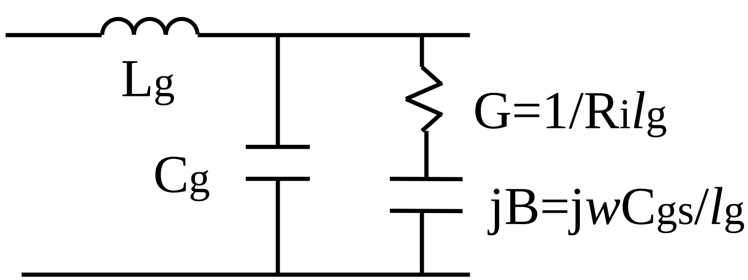






Discussion

1. unit cell of gate line



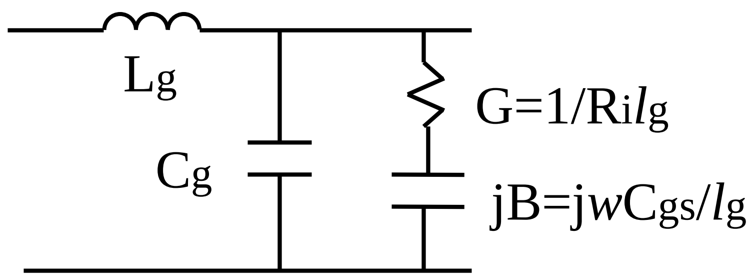
$$Z_g \approx \sqrt{\frac{L_g}{C_g + C_{gs}/l_g}}$$

$$\gamma_g = \sqrt{jwL_g\left(jwC_g + \frac{jwC_{gs}/l_g}{1+jwR_iC_{gs}}\right)}$$

$$\underset{wR_iC_{gs}<<1}{\overset{\text{small loss}}{\approx}} \frac{w^2R_iZ_gC_{gs}^2}{2l_g} + jw\sqrt{L_g\left(C_g + \frac{C_{gs}}{l_g}\right)}$$

$$= \alpha_g + j\beta_g$$

(derivation of 1)



$$Z = jwL_g, Y = jwC_g + \frac{jwC_{gs}/l_g}{1+jwR_iC_{gs}}$$

$$Z_g = \sqrt{\frac{Z}{Y}} \underset{wR_iC_{gs}<<1}{\overset{\text{small loss}}{\approx}} \sqrt{\frac{L_g}{C_g + C_{gs}/l_g}}$$

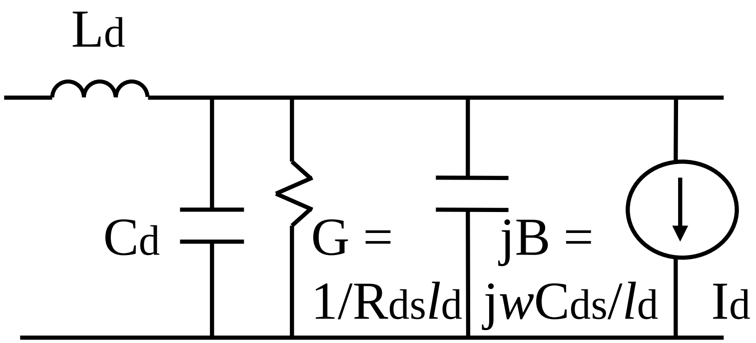
$$\gamma_g = \sqrt{ZY} = \sqrt{jwL_g\left(jwC_g + \frac{jwC_{gs}/l_g}{1+jwR_iC_{gs}}\right)} \overset{wR_iC_{gs}<<1}{\approx} \sqrt{jwL_g\left[jwC_g + \frac{jwC_{gs}(1-jwR_iC_{gs})}{l_g}\right]}$$

$$= \sqrt{(jw)^2L_g\left(C_g + \frac{C_{gs}}{l_g}\right) - \frac{(jw)^3L_gR_iC_{gs}^2}{l_g}}$$

$$\overset{(a-b)^{\frac{1}{2}} \approx a^{\frac{1}{2}} - \frac{1}{2}a^{-\frac{1}{2}}b}{\approx} \sqrt{(jw)^2L_g\left(C_g + \frac{C_{gs}}{l_g}\right)} - \frac{1}{2}\frac{(jw)^3L_gR_iC_{gs}^2/l_g}{jw\sqrt{L_g(C_g + C_{gs}/l_g)}}$$

$$= jw\sqrt{L_g\left(C_g + \frac{C_{gs}}{l_g}\right)} + \frac{w^2}{2}\sqrt{\frac{L_g}{C_g + \frac{C_{gs}}{l_g}}}\frac{R_iC_{gs}^2}{l_g} = jw\sqrt{L_g\left(C_g + \frac{C_{gs}}{l_g}\right)} + \frac{w^2R_iC_{gs}^2Z_g}{2l_g} = \alpha_g + j\beta_g$$

2. unit cell of drain line



$$Z_d \approx \sqrt{\frac{L_d}{C_d + C_{ds}/l_d}}$$

$$\gamma_d = \sqrt{jwL_d[\frac{1}{R_{ds}l_d} + jw(C_d + \frac{C_{ds}}{l_d})]}$$

small loss

$$\approx \frac{Z_d}{2R_{ds}l_d} + jw\sqrt{L_d(C_d + \frac{C_{ds}}{l_d})}$$

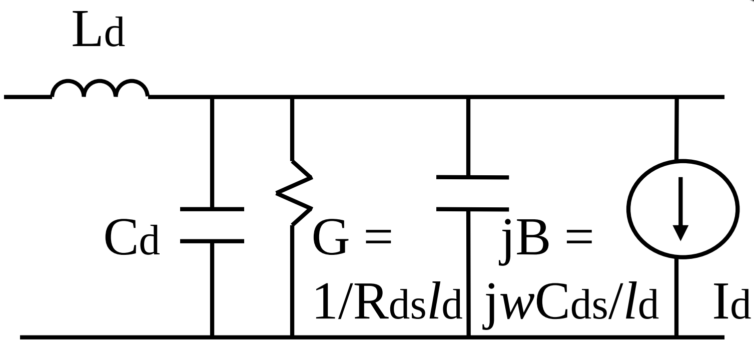
$$= \alpha_d + j\beta_d$$

3. o/p current

$$I_o = -\frac{1}{2}\sum_{n=1}^{N} I_{dn} e^{-(N-n)\gamma_d l_d}, I_{dn} = g_m V_{cn}, V_{cn} = V_i e^{-(n-1)\gamma_g l_g}(\frac{1}{1+jwR_i C_{gs}})$$

$$\rightarrow I_o = -\frac{g_m V_i}{2} e^{-N\gamma_d l_d} e^{\gamma_g l_g} \sum_{n=1}^{N} e^{-n(\gamma_g l_g - \gamma_d l_d)} = -\frac{g_m V_i}{2} \frac{e^{-N\gamma_g l_g} - e^{-N\gamma_d l_d}}{e^{-\gamma_g l_g} - e^{-\gamma_d l_d}}$$

(derivation of 2)



$$Z = jwL_d, Y = \frac{1}{R_{ds}l_d} + jw(C_d + \frac{C_{ds}}{l_d})$$

$$Z_d = \sqrt{\frac{Z}{Y}} \underset{R_{ds}l_d<<1}{\overset{\text{small loss}}{\approx}} \sqrt{\frac{L_d}{C_d + C_{ds}/l_d}}$$

$$\gamma_d = \sqrt{ZY} = \sqrt{jwL_d[\frac{1}{R_{ds}l_d} + jw(C_d + \frac{C_{ds}}{l_d})]} = \sqrt{(jw)^2 L_d(C_d + \frac{C_{ds}}{l_d}) + \frac{jwL_d}{R_{ds}l_d}}$$

$$\overset{(a+b)^{\frac{1}{2}} \approx a^{\frac{1}{2}} + \frac{1}{2}a^{-\frac{1}{2}}b}{\approx} jw\sqrt{L_d(C_d + \frac{C_{ds}}{l_d})} + \frac{1}{2}\frac{1}{\sqrt{(jw)^2 L_d(C_d + C_{ds}/l_d)}}\frac{jwL_d}{R_{ds}l_d}$$

$$= jw\sqrt{L_d(C_d + \frac{C_{ds}}{l_d})} + \frac{1}{2}\frac{L_d}{R_{ds}l_d}\frac{1}{\sqrt{L_d(C_d + C_{ds}/l_d)}} = jw\sqrt{L_d(C_d + \frac{C_{ds}}{l_d})} + \frac{1}{2}\frac{1}{R_{ds}l_d}\sqrt{\frac{L_d}{C_d + C_{ds}/l_d}}$$

$$= jw\sqrt{L_d(C_d + \frac{C_{ds}}{l_d})} + \frac{1}{2}\frac{Z_d}{R_{ds}l_d} = \alpha_d + j\beta_d$$

(derivation of 3)

$$I_o = -\frac{1}{2}\sum_{n=1}^{N} I_{dn} e^{-(N-n)\gamma_d l_d}, I_{dn} = g_m V_{cn}, V_{cn} = V_i e^{-(n-1)\gamma_g l_g}\left(\frac{1}{1+jwR_i C_{gs}}\right)$$

$$\rightarrow I_o = -\frac{g_m}{2}\sum_{n=1}^{N} V_{cn} e^{-(N-n)\gamma_d l_d} = -\frac{g_m V_i}{2} e^{-N\gamma_d l_d} e^{\gamma_g l_g}\sum_{n=1}^{N} e^{-n(\gamma_g l_g - \gamma_d l_d)}, \sum r^n = \frac{r(1-r^N)}{1-r}$$

$$= -\frac{g_m V_i}{2} e^{-N\gamma_d l_d} e^{\gamma_g l_g} \frac{e^{-(N+1)(\gamma_g l_g - \gamma_d l_d)} - e^{-(\gamma_g l_g - \gamma_d l_d)}}{e^{-(\gamma_g l_g - \gamma_d l_d)} - 1} \times \frac{e^{-\gamma_d l_d}}{e^{-\gamma_d l_d}}$$

$$= -\frac{g_m V_i}{2} e^{-(N+1)\gamma_d l_d} e^{\gamma_g l_g} \frac{e^{-(N+1)(\gamma_g l_g - \gamma_d l_d)} - e^{-(\gamma_g l_g - \gamma_d l_d)}}{e^{-\gamma_g l_g} - e^{-\gamma_d l_d}}$$

$$= -\frac{g_m V_i}{2} \frac{e^{-N\gamma_g l_g} - e^{-N\gamma_d l_d}}{e^{-\gamma_g l_g} - e^{-\gamma_d l_d}}$$

4. For matched i/p and o/p ports

$$G=\frac{P_{out}}{P_{in}}=\frac{\dfrac{|I_o|^2 Z_d}{2}}{\dfrac{|V_i|^2}{2Z_g}}=\frac{|I_o|^2 Z_d Z_g}{|V_i|^2}=\frac{g_m^2 Z_d Z_g}{4}\left|\frac{e^{-N\gamma_g l_g}-e^{-N\gamma_d l_d}}{e^{-\gamma_g l_g}-e^{-\gamma_d l_d}}\right|^2$$

$$=\frac{g_m^2 Z_d Z_g}{4}\left|\frac{e^{-N(\alpha_g l_g+j\beta_g l_g)}-e^{-N(\alpha_d l_d+j\beta_d l_d)}}{e^{-(\alpha_g l_g+j\beta_g l_g)}-e^{-(\alpha_d l_d+j\beta_d l_d)}}\right|^2$$

under synchronization condition $\beta_g l_g=\beta_d l_d(\theta_g=\theta_d)$

$$\rightarrow G=\frac{g_m^2 Z_d Z_g}{4}\frac{(e^{-N\alpha_g l_g}-e^{-N\alpha_d l_d})^2}{(e^{-\alpha_g l_g}-e^{-\alpha_d l_d})^2}, N\Uparrow\infty\Rightarrow G\Downarrow 0$$

$$\frac{dG}{dN}=0\rightarrow N_{opt}=\frac{\ln(\alpha_g l_g/\alpha_d l_d)}{\alpha_g l_g-\alpha_d l_d}$$

For a lossless amplifier (R_i =0, $R_{ds}=\infty$) and if $Z_d=Z_g=Z_o$

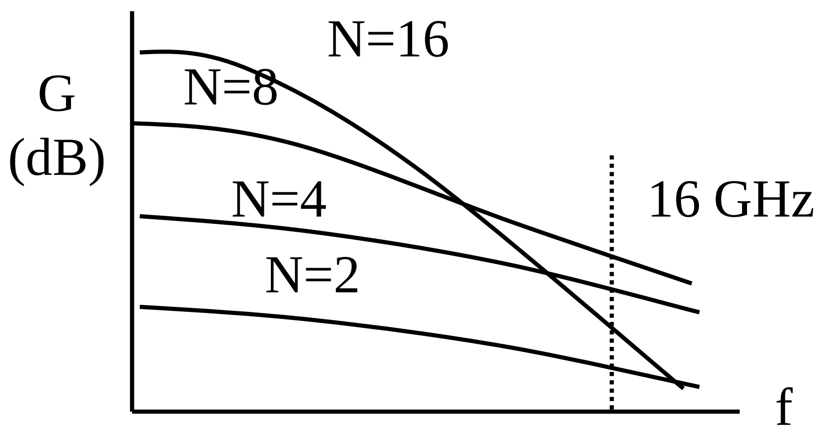
$$\rightarrow G=\frac{g_m^2 Z_d Z_g N^2}{4}=\left(\frac{g_m Z_o N}{2}\right)^2, G\propto N^2$$

5. Ex.12.8 $Z_d= Z_g = Z_o=50\Omega$, $R_i=5\ \Omega$, $R_{ds}=250\ \Omega$, $C_{gs}=0.3pF$, $g_m=30mS$

$$\alpha_g l_g = \frac{w^2 R_i C_{gs}^2 Z_o}{2} = 0.1 @ 16GHz$$

$$\alpha_d l_d = \frac{Z_o}{2R_{ds}} = 0.114 @ 16GHz$$

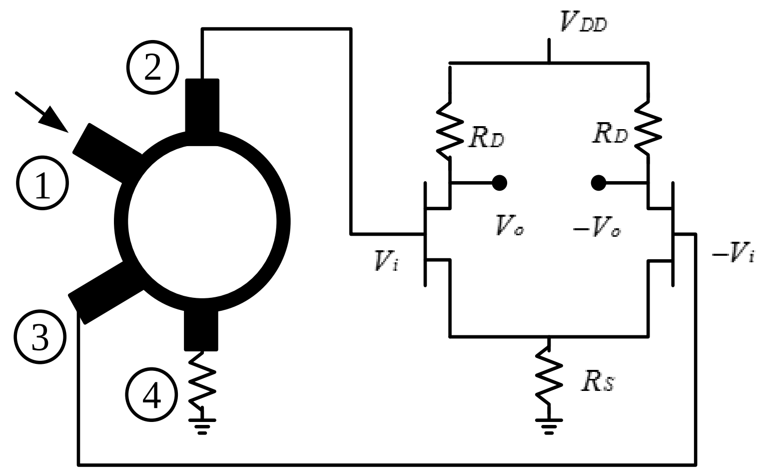
$\rightarrow N_{opt} = 9.4,$ frequency response (p.593, Fig.12.16)

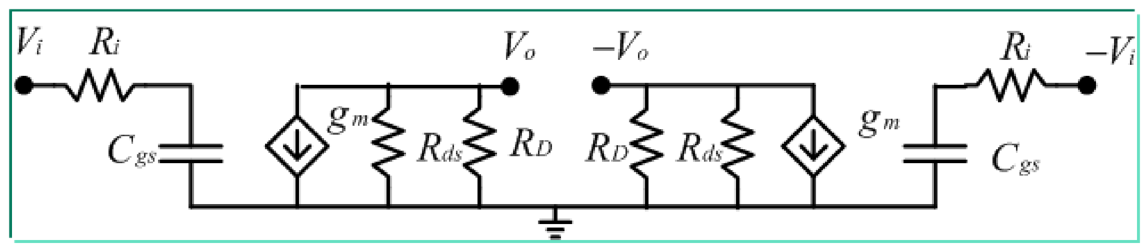


- Differential amplifier

$$\frac{-j}{\sqrt{2}}\begin{bmatrix}0 & 1 & 1 & 0\\ 1 & 0 & 0 & -1\\ 1 & 0 & 0 & 1\\ 0 & -1 & 1 & 0\end{bmatrix}\begin{bmatrix}0\\0\\0\\a\end{bmatrix}=\begin{bmatrix}0\\ \frac{-ja}{\sqrt{2}}\\ \frac{ja}{\sqrt{2}}\\ 0\end{bmatrix}$$

balun





$$V_{gs}=\frac{V_i}{1+j\omega R_i C_{gs}}$$

$$V_o=-g_m V_{gs}\frac{R_D R_{ds}}{R_D+R_{ds}}=-\frac{g_m R_D R_{ds}}{(1+j\omega R_i C_{gs})(R_D+R_{ds})}V_i$$

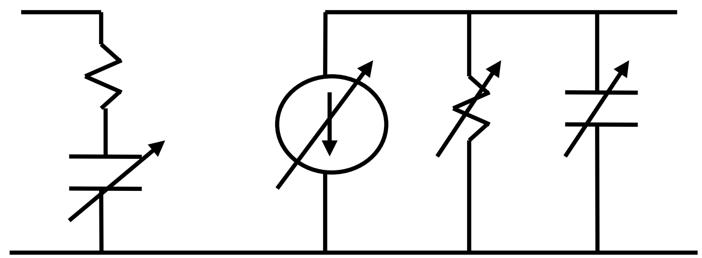
$$A_d=\frac{V_o-(-V_o)}{V_i-(-V_i)}=\frac{g_m R_D R_{ds}}{(1+j\omega R_i C_{gs})(R_D+R_{ds})}$$

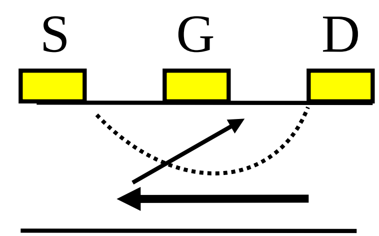
output swing and f_T doubled

11.5 Power amplifiers

• nonlinear operation→S(input power, f, DC, T, Z_L)

FET nonlinear equivalent circuit (large-signal S-parameter)



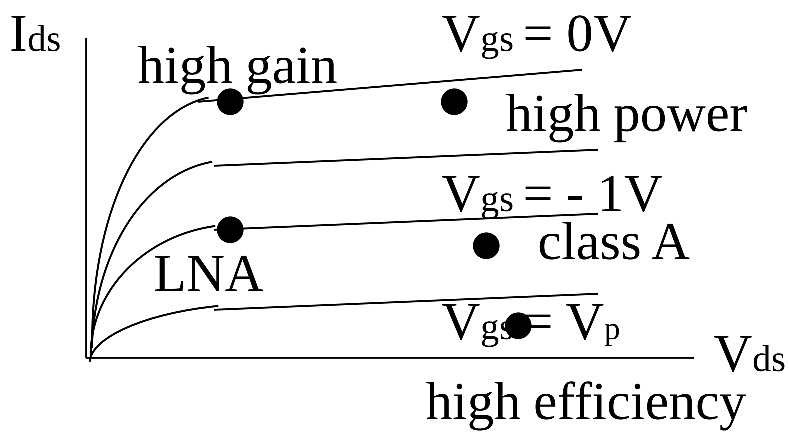


Discussion

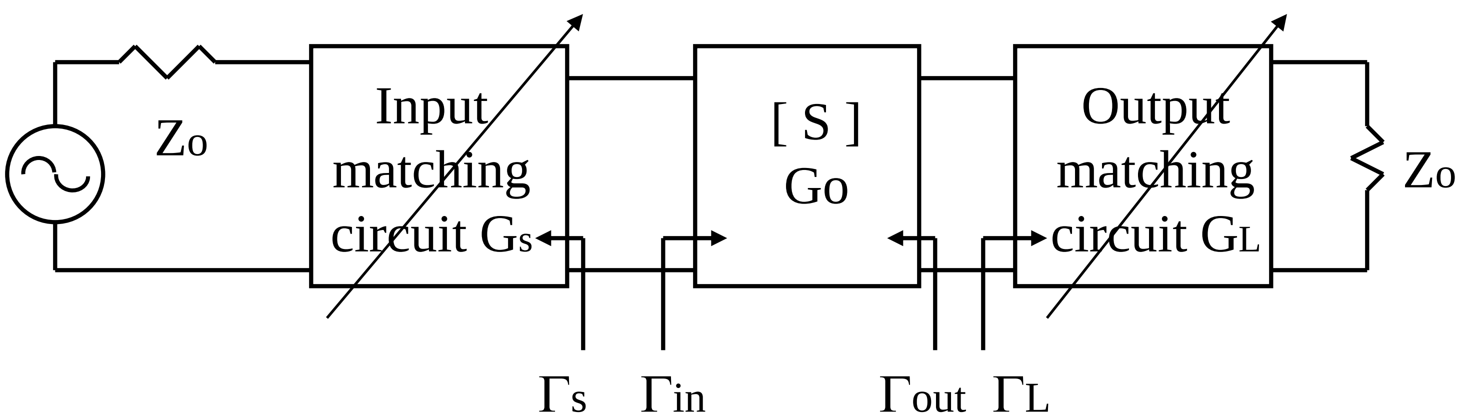
1. power amplifier characteristics: efficiency, gain, intermodulation product, thermal conduction

power added efficiency $$\eta_{\text{PAE}} \equiv \frac{P_{out} - P_{in}}{P_{DC}}$$

2. DC bias consideration



3. design consideration: large-signal source impedance Γ_s (source-pull contour) and load impedance Γ_L (load-pull contour)



4. Ex.12.9 a transistor has small-signal S-parameters at 2.3GHz as

$S_{11} = 0.593\angle 178°, S_{12} = 0.009\angle -127°, S_{21} = 1.77\angle -106°, S_{22} = 0.958\angle 175°$
For class A operation at $V_{DS} = 28V$ and $I_D = 0.6A, P_o = 10W, G = 16.4dB,$
$Z_{SP} = 10 - j3\Omega, Z_{LP} = 2.5 + j2.3\Omega,$ design the input and output matching circuits.

From small-signal S-parameter, $|\Delta| = 0.579 < 1, K = 2.08 > 1 \rightarrow$ unconditional stable

From Z_{SP} and $Z_{LP} \rightarrow \Gamma_{SP} = 0.668\angle 187°, \Gamma_{LP} = 0.905\angle -175°$

From small-signal S-parameter for $G_{T\max} \rightarrow \Gamma_S = 0.508\angle 166°, \Gamma_L = 0.954\angle -176°$

For $P_{out} = 10W, P_{in} = P_{out} - G = 23.6dBm = 229mW$

$$\eta_{PAE} = \frac{P_{out} - P_{in}}{VI} = \frac{10 - 0.229}{28 \times 0.6} = 25\%$$

<u>ADS examples</u>: Ch12_prj