Microwave Amplifiers

Design of Microwave Transistor Amplifiers Using S Parameters

Microwave amplifiers combine active elements with passive transmission line circuits to provide functions critical to microwave systems and instruments. The history of microwave amplifiers begins with electron devices using resonant or slow-wave structures to match wave velocity to electron beam velocity.

The design techniques used for BJT and FET amplifiers employ the full range of concepts we have developed in the study of microwave transmission lines, two-port networks and Smith chart presentation.

The development of S-parameter matrix concepts grew from the need to characterize active devices and amplifiers in a form that recognized the need for matched termination rather than short- or open-circuit termination. Much of the initial work was performed at the Hewlett-Packard Company in connection with the development of instruments to measure device and amplifier parameters.

We'll begin by considering microwave amplifiers that are

1) Small signal so that superposition applies, and
2) Built with microwave bipolar junction or field-effect transistors

References

The following books and notes are references for this material:

Pozar[1], D. M., *Microwave Engineering*

Gonzalez[2], G., *Microwave Transistor Amplifiers*

Vendelin, Pavio & Rohde[3], *Microwave Circuit Design Using Linear and Nonlinear Techniques*

Review of Transmission Lines

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1 Pozar, D., *Microwave Engineering*, 2nd Edition, J. Wiley, 1998, pg. 600-640
2 Gonzalez, G., *Microwave Transistor Amplifiers,*
3 Vendelin, Pavio & Rohde, *Microwave Circuit Design Using Linear and Nonlinear Techniques*, J. Wiley, 1990

For the purpose of characterizing microwave amplifiers, key transmission line concepts are

1) Traveling waves in both directions, V^+ and V^-
2) Characteristic impedance Z_o and propagation constant jß
3) Reflection coefficient $\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$ for complex load Z_L
4) Standing waves resulting from $\Gamma \neq 0$
5) Transformation of Z_L through line of Z_o and length ßl
6) Description of Γ and Z on the Smith chart (polar graph of Γ)

<u>Review of Scattering Matrix</u>

1) Normalization with respect to $\sqrt{Z_o}$ of wave amplitudes:

$$a = \frac{V^+}{\sqrt{Z_o}} \text{ and } b = \frac{V^-}{\sqrt{Z_o}}$$

2) Relationship of b_i and a_i: $b_i = \Gamma_i a_i$
3) Expressions for b_1 and b_2 at reference planes:

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

4) Definitions of S_{ii}:

$S_{11} = \frac{b_1}{a_1}$ for $a_2 = 0$, i.e., input Γ for output terminated in Z_o.

$S_{21} = \frac{b_2}{a_1}$ for $a_2 = 0$, i.e., forward transmission ratio with Z_o load.

$S_{22} = \frac{b_2}{a_2}$ for $a_1 = 0$, i.e., output Γ for input terminated in Z_o.

$S_{21} = \frac{b_1}{a_2}$ for $a_1 = 0$, i.e., reverse transmission ratio with Z_o source.

$|S_{21}|^2$ = Transducer power gain with Z_o source and load.

5) Definitions of Γ_L, Γ_s, Γ_{in} and Γ_{out}:

$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$, the reflection coefficient of the load

$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o}$, the reflection coefficient of the source

$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$, the input reflection coefficient

$\Gamma_{out} = \frac{Z_{out} - Z_o}{Z_{out} + Z_o} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$, the output reflection coefficient

6) Power Gain G, Available Gain G_A, Transducer Gain G_T:

$$G = \frac{P_L}{P_{in}} = \frac{\text{power delivered to the load}}{\text{power input to the network}}$$

$$G_A = \frac{P_{av_{out}}}{P_{av_s}} = \frac{\text{power available from the network}}{\text{power available from the source}}$$

$$G_T = \frac{P_L}{P_{av_s}} = \frac{\text{power delivered to the load}}{\text{power available from the source}}$$

Modeling of Microwave Transistors and Packages

The S parameters of a given microwave transistor can be derived from transistor equivalent circuit models based on device physics, or they can be measured directly. Generally, a manufacturer of a device intended for microwave applications will provide extensive S-parameter data to permit accurate design of microwave amplifiers. This can be verified by measurement, a step that has proven important on many occasions.

For a bipolar junction transistor, in addition to intrinsic device parameters such as base resistance and collector-base capacitance, amplifier performance is strongly affected by the so-called parasitic elements associated with the device package, including base-lead and emitter-lead inductance internal to the package. Similar considerations apply to microwave field-effect transistors.

The magnitude and phase angle of each of the S parameters typically vary with frequency, and characterization over the complete range of interest is necessary.

An abridged table from Pozar[4] typical of S-parameter data (Z_o=50Ω) is shown here for a microwave FET:

f GHz	S_{11}	S_{21}	S_{12}	S_{22}
3.0	$0.80\underline{/-89°}$	$2.86\underline{/99°}$	$0.03\underline{/-56°}$	$0.76\underline{/-41°}$
4.0	$0.72\underline{/-116°}$	$2.60\underline{/76°}$	$0.03\underline{/-57°}$	$0.73\underline{/-54°}$
5.0	$0.66\underline{/-142°}$	$2.39\underline{/54°}$	$0.03\underline{/-62°}$	$0.72\underline{/-68°}$

Power Gain Equations

The equations for the various power gain definitions are

[4] Pozar, D., *Microwave Engineering*, 2nd Edition, J. Wiley, 1998, pg. 620

1) $$G = \frac{P_L}{P_{in}} = \frac{1}{1 - |\Gamma_{in}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

2) $$G_A = \frac{P_{av_{out}}}{P_{av_s}} = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_{out}|^2}$$

3) $$G_T = \frac{P_L}{P_{av_s}} = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{in}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$
$$= \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{out}\Gamma_L|^2}$$

The expressions for Γ_{in} and Γ_{out} are

1) $$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

2) $$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

For a unilateral network, $S_{12}=0$ and

1) $\Gamma_{in} = S_{11}$ if $S_{12}=0$ (unilateral network)

2) $\Gamma_{out} = S_{22}$ if $S_{12}=0$ (unilateral network)

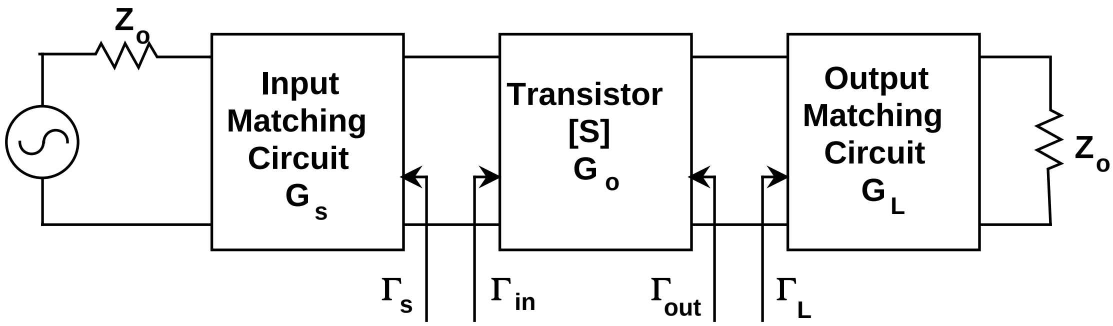
The transducer gain G_T can be expressed as the product of three gain contributions

$G_T=G_sG_oG_L$, where

$G_o = |S_{21}|^2$

$$G_s = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{in}\Gamma_s|^2}$$ and

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$



If the device is unilateral, or sufficiently so that S_{12} is small enough to be ignored, the unilateral transducer gain G_{TU} is simplified because

$G_{sU} = \dfrac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2}$, where the subscript U indicates unilateral gain.

In practice, the difference between G_T and G_{TU} is often quite small, as it is desirable for devices to be unilateral if possible.

The components of G_{TU} can also be expressed in decibel form, so that

G_{TU} (dB) = G_s (dB) + G_o (dB) + G_L (dB).

We can maximize G_s and G_L by setting $\Gamma_s = S_{11}^*$ and $\Gamma_L = S_{22}^*$ so that

$G_{s_{max}} = \dfrac{1}{1 - |S_{11}|^2}$ and

$G_{L_{max}} = \dfrac{1}{1 - |S_{22}|^2}$, so that

$$G_{TU_{max}} = \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2}$$

Note that, if $|S_{11}|=1$ or $|S_{22}|=1$, $G_{TU_{max}}$ is infinite. This raises the question of stability, which will be examined next.

Stability

In a two-port network, oscillations are possible if the magnitude of either the input or output reflection coefficient is greater than unity, which is equivalent to presenting a negative resistance at the port. This instability is characterized by

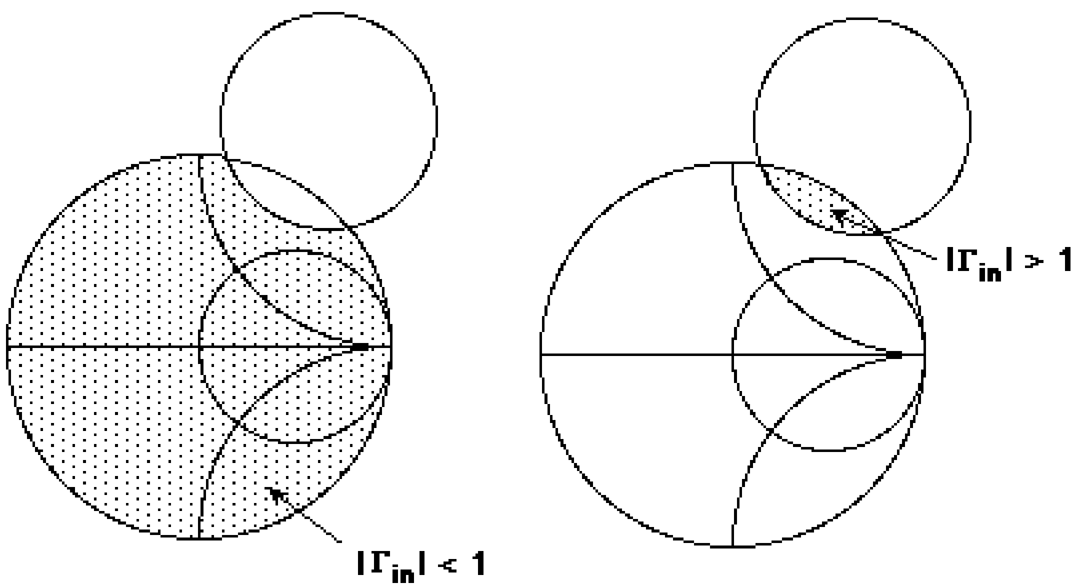
$|\Gamma_{in}| > 1$ or $|\Gamma_{out}| > 1$, which for a unilateral device implies $|S_{11}| > 1$ or $|S_{22}| > 1$.

Thus the requirements for stability are

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1 \text{ and}$$

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| < 1$$

These are defined by circles, called stability circles, that delimit $|\Gamma_{in}| = 1$ and $|\Gamma_L| = 1$ on the Smith chart. The radius and center of the output and input stability circles are derived from the S parameters on pg. 614 of Pozar or pg. 97 of Gonzalez. The concept of instability with varying input or output matching conditions is significant, as we would desire an amplifier to be unconditionally stable under all expected conditions of source and load impedances. The example of input stability circles is shown here.



This same derivation can be accomplished analytically. The conditions for stability are

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 - |S_{12}S_{21}|} > 1 \text{ and}$$

$|\Delta| < 1$, where Δ, the determinant of the scattering matrix, is

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

If an amplifier is conditionally stable, it can be rendered unconditionally stable by adding resistance to the input and/or output of the amplifier so that the total loop resistance at the input and output is positive. The use of resistive loading or feedback can compromise the noise performance of an amplifier unless accomplished in connection with an analysis of the amplifier noise figure.

Constant Gain Circles

Because of the form of the unilateral transducer gain, values of the input reflection coefficient Γ_{in} that produce constant gain also lie on circles on the Smith chart. The derivation of the radius and center of these circles is found on pg. 621-626 of Pozar and pp. 102-105 of Gonzalez.

Noise in Amplifiers

The lower limit of amplifier signal capability is set by noise. Three sources of noise in transistor amplifiers are

1) Thermal noise due to random motion of charge carriers due to thermal agitation: Available noise power P_{av}=kTB.
2) Shot noise due to random flow of carriers across a junction, which produces a noise current of $i_n^2 = 2qI_{dc}B$
3) Partition noise due to recombination in the junction, which produces a noise current of $i_p^2 = \frac{2kT}{r_e'} \alpha_o(1 - \alpha_o) B$

In these expressions, kT=-174 dBm in 1 Hz bandwidth, B is the bandwidth, q is the charge of the electron, and the other parameters are elements of the transistor equivalent circuit.

The noise figure F of an amplifier is defined as the ratio of the total available noise power at the output of the amplifier to the available noise power at the output that would result only from the thermal noise in the source resistance. Thus F is a measure of the excess noise added by the amplifier. Amplifier noise can also be characterized by an equivalent noise temperature of the source resistance that would provide the same available noise power output. This equivalent noise temperature is given by

$$T_e=(F - 1)T_o$$

Because of the interaction of the various noise sources and resistances of a microwave transistor, the noise figure of an amplifier generally varies as

$$F = F_{min} + \frac{R_N}{G_S} |Y_s - Y_{opt}|^2, \text{ where}$$

Y_s is the source admittance presented to the transistor
Y_{opt} is the optimum source admittance that results in minimum noise figure
F_{min} is the minimum noise figure
R_N is the equivalent noise resistance of the transistor, and
G_s is the real part of Y_s

If instead of admittance we use reflection coefficients, it will come as no surprise that we can find circles of constant noise figure on the Smith chart, as derived in Pozar on pg. 629.
We can generally expect to be provided with F_{min}, Γ_{opt} and R_N for a given device and frequency. These parameters can, of course, be derived also from direct measurement of noise figure under conditions of optimum source impedance. It is unusual for noise figure and gain circles to be concentric, as maximum gain conditions are not the same as minimum noise figure conditions.

The noise figure of cascaded amplifiers is given by the numerical (not dB) relationship

$$F = F_1 + \frac{F_2 - 1}{G_{A1}} ,$$

where F_1 and G_{A1} are the noise figure and available gain of the first stage, and F_2 is the noise figure of the second stage. This applies to lossy stages and networks as well.

Power Amplifiers

The design of power amplifiers involves less emphasis on noise parameters, and more emphasis on linearity and intermodulation, as well as efficiency and thermal considerations. To design a power amplifier, one must use large-signal S-parameters and be aware of nonlinear effects.

Where careful design of the input matching network is required to realize the full capabilities of low-noise amplifiers, in power amplifiers more emphasis tends to be on optimizing the output matching network. There are, however, special problems associated with the very low input impedance that can be found in bipolar power devices, which require special treatment in the input matching network if wideband operation is to be achieved.

A key issue for multi-stage amplifiers is the ability to cascade individually designed stages without a requirement for retuning or redesign to account for the characteristics of the driving or following stages. In many cases, the use of balanced amplifiers permits the benefit of 3 dB coupler interstages, which direct reflected power to the isolated port rather than the driving stage. As we will see in later lectures, there are special problems of nonlinear oscillations arising from interaction between signal harmonics and modes of the output matching structure.

Impedance Matching with Microstrip Lines

Input and output circuit impedance matching can be accomplished with simple lumped-element networks, or with the equivalent short lengths of transmission line. The required element values can be determined by use of the Smith chart or by calculations using a computer.

Let's see how to get a series inductor or shunt capacitor using short sections of transmission line. Recall that for a short length of transmission line of characteristic impedance Z_o and velocity v, the inductance L per unit length (same units as v) is given by

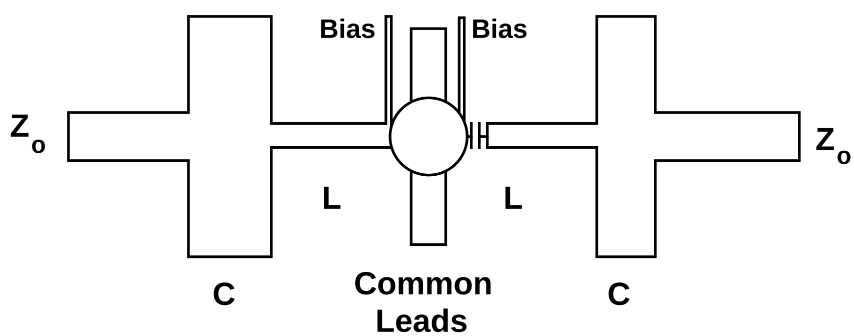
$$\frac{L}{d} = \frac{Z_o}{v}$$, where

$$v = c/\sqrt{\varepsilon_{eff}}$$

For a capacitive line (short open line), C per unit length d is

$$\frac{C}{d} = \frac{1}{Z_o v}$$

We need to correct for the fringing capacitance, which should be subtracted from the capacitance value desired from the transmission line[5].



Incorrect treatment of common lead grounding is a major cause of reduced gain or instability in microwave amplifiers. In some cases, plated-through holes are used to carry the ground plane of the microstrip up to the common leads, but this will result in common-lead inductance unless the microstrip is quite thin (< 1 mm). Other methods include mounting the active device in a hole, with the common leads soldered to the ground side of the board.

[5] Silvester and Benedek, "Equivalent Capacitances of Microstrip Open Circuits," *IEEE Trans. Microwave Theory*, Vol. MTT-20, pp. 511-516, Aug. 1972

Bias Circuits and Bias Circuit Instabilities

Once the microwave amplifier is designed, it remains to provide the dc bias voltages and currents required for the active device. This is no simple problem, as the arrangements to introduce the biases can disturb the microwave circuit. Generally, high impedance microstrip traces can be used as decoupling inductors, but caution must be exercised not to create a low frequency oscillator circuit in the bias network.

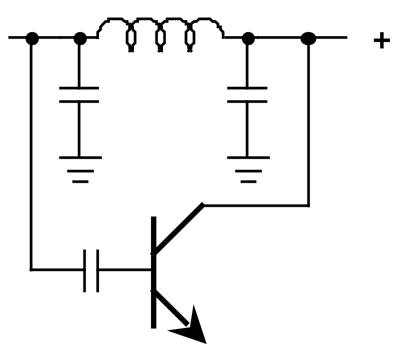
A common cause of trouble is the use of an inductor with a large bypass capacitor, which can create a resonator in the MHz region that can support oscillation of the active element, which will have very high gain at lower frequencies.

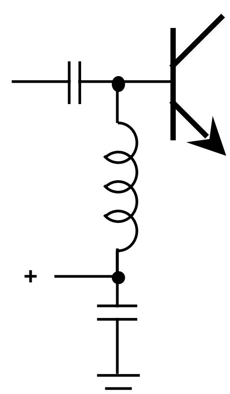
Bias-circuit instabilities are a common source of problems in amplifiers and other active circuits. These generally result from the use of inductors and capacitors in the bias circuit without regard to resonances or situations where 180° phase shift can occur.

Two examples of circuit configurations that can promote bias oscillations at low frequencies are

- The use of an inductor with bypass capacitors on both sides to filter dc supply to earlier stages of an amplifier; this can have 180° phase shift at a video frequency where the active element has substantially more gain than at microwave frequencies.
- The use of an inductor with bypass capacitor to isolate dc supply input to the base or gate of the active element; this can form a resonator for video frequency oscillations.

Examples of bias circuits that are prone to parasitic oscillation are shown here:





In the circuit on the left, the potential for 180° phase shift across the inductor can be seen. The cure is to use inductors that are lossy, either because of resistive loading or by use of lossy ferrite cores to provide RF isolation. The circuit on the right exhibits a resonant circuit that can result in parasitic oscillation; in this case, the small current required for base or gate bias permits the use of resistance in series with the inductor. It is possible to use a wire-wound resistor to accomplish both functions at the same time.

It is generally helpful to sketch the low-frequency equivalent circuit to be sure that no instabilities can exist. Resistive or lossy elements are required to guarantee stability in the bias circuit.

In the past, emphasis was placed on achieving the minimum number of components for reasons of cost and reliability. With the introduction of integrated circuits, this concept has been superseded, and active feedback bias circuits are generally used to insure the stability of device operating conditions.