The study of fast adaptive algorithms and introducing new methods for increasing the rate of convergence and its use in smart antennas

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Abstract: *This paper, the algorithms introduced for increasing the rate of convergence in which for its achievement the rates of convergence in adaptive algorithms are studied and new methods for selection of μ step-size are given. In this methods especial function are introducing for the definition of μ which overturn current limitations in selection of μ and increase the rate of convergence and decrease noise.*

Keywords: Adaptive filters, rate of convergence, covariance matrix, adaptive algorithms, smart antenna.

1 Introduction

To have need of high rates in variable usage of adaptive filters make it important to care seriously to the use of new algorithms with high rate. In recent years new adaptive systems with stable and unstable inputs are known. In most of these, high rate of convergence is needed. Some of the usages of these systems are followed as [3]

- Noise cancelling
- Tracking radars considering aims speed highly increased now days
- Echo cancellers in a measurement room
- Smart antenna
- Automatic equalizers in digital cellular radio systems which exhibit rapid fading problems

and other systems related to the control. The main aim of increasing rate in posed fields is that by change of filters weights can easily chase fast input fluctuations. However, increasing the rate causes, decreasing of power use and fast setting in most of adaptive filters usages especially in smart antennas. Primary use of filter is to find a W weight vector which with its use on the vector of input signal, its exhaust to be near enough to desired signal, and this is done by different algorithm before [1]. For getting this result, the received signal's input correlation matrix should be evaluated as

$$R = E\left[X_k X_k^T\right] \qquad (1)$$

Where E shows expectation and X_k is vector of input signal.

For getting to high speeds many tasks such as selection of variable step-size and or use of new algorithms have done [3]. In this paper, the limitations of primary algorithms of adaptive filters and papers which are relevant to this topic are reviewed. In these algorithms different problems are protruding for the convergence of the weights which are concern with the amount of step-size. Some of these problems are [1, 2]

1- μ step-size should be selected in a way that weights don't get in fluctuation convergence and unstable domain.

2- The range of selection of μ step-size is limited.

By considering these a new method for increasing the speed sufficiently without increasing the noise fluctuation is given and its results are shown in mathematic relationships and comparison of simulation results with other algorithms.

2 Newton's algorithm

Newton's algorithm is [1].

$$W_{k+1} = (1-2\mu)W_k + 2\mu W^* \qquad (2)$$

or

$$W_k = W^* + (1-2\mu)^k (W_0 - W^*) \qquad (3)$$

where W^* is optimum weight vector, W_0 is initial weight vector, W_k is the weight vector of kth iteration.

$$W^* = R^{-1}P \qquad (4)$$

$$W_k = [w_1, w_2, ..., w_n] \qquad (5)$$

where R is input correlation matrix and P is cross correlations between the desired response and the input components.

$$P = E[d_k X_k^T] \qquad (6)$$

where X_k is input vector and d_k is desired response.

$$X_k = [x_k, x_{k-1}, ..., x_{k-n+1}] \qquad (7)$$

where n is the number of coefficients.

$$0 \prec \mu \prec 1 \qquad (8)$$

If the condition in (8) is met, that is, if the algorithm in (2) or (3) is stable, the algorithm is seen to converge to the optimum solution.

3 Modified algorithms of Newton's method

3.1 First method

With some changes in choosing μ a modified Newton's algorithm can be introduce.

$$W_k = W^* + (1-2m)^{fk} (W_0 - W^*) \qquad (9)$$

Where f is a factor got by proper selection of μ and for preventing from fluctuation convergence of weights, f is selected as an integer and even number. By selection of stable range for μ in fact between zero and one, the rate of convergence will considerably increase by increasing of f. For making Eq. (9), the steps are processed in below from in which it is first considered.

$$(1-2\mu) = (1-y)^f \qquad (10)$$

that

$$(1-y)^f = 1 - \sum_{h=1}^{h=f} \frac{(-1)^{h+1} f!}{h!(f-h)!} y^h \qquad (11)$$

By comparison of (10) and (11)

$$\mu = 0.5 \sum_{h=1}^{h=f} \frac{(-1)^{h+1} f!}{h!(f-h)!} y^h \qquad (12)$$

After finding y, it can be written in (13) form so that it can be contrasted with Newton's algorithm.

$$y = 2m \qquad (13)$$

By choosing μ from Eq. (12) and substituting it in Eq. (2), the algorithm of Eq. (9) is obtained. Concerning to modified algorithm and its comparison with Newton's algorithm, the rate of convergence will be considerably increased. If f is considered even, weights do not arrive in fluctuation convergence. In modified Newton's algorithm, for the convergence of adaptive filter, m should be between zero and one (stable range). To compare the rate of convergence of the new method, an identification system in consider Figure 1 and the results are shown in Figure 2.

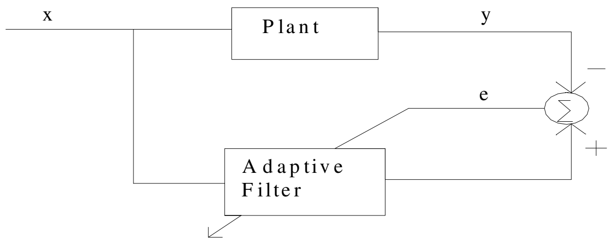


Figure 1: The identification system.

Figure 1 shows the block diagram of adaptive filter usage for identification of unknown plant in which the filter try to change weights and approach to unknown plant character and minimize error. In Figure 2 the simulation of this example whit μ=0.01, m=0.01 and W_0 =50 are shown.

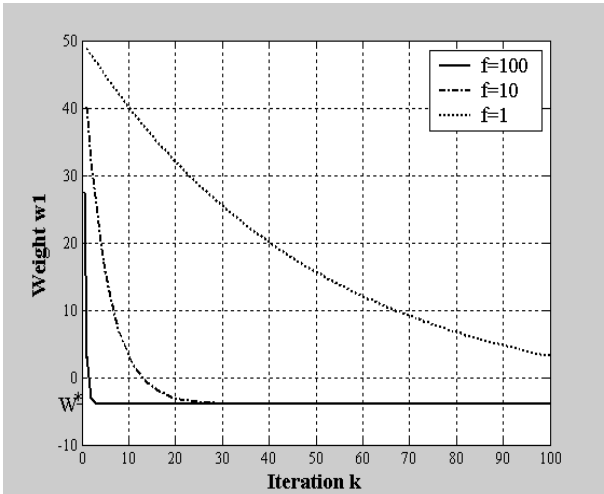


Figure 2: Filter weights in accordance with different value of f.

Figure 2 shows the rate of convergence of adaptive filter weights to W^*. Graph f=1 is related to the result of Newton's algorithm and graphs of f=10 and f=100 are the results of the modified algorithm. As it is seen, the rate of convergence is considerably increased. In Figure 3 the simulation of μ=0.51, m=0.51 and W_0 =50 is shown.

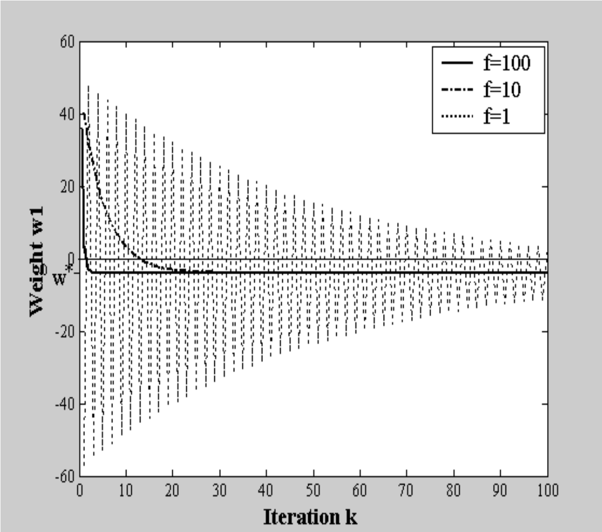


Figure 3: filter weights in accordance with different value of f.

Figure 3 shows the rate of convergence of adaptive filter weights to W^*. Graph f=1 is related to the result of Newton's algorithm and in accordance with step-size of range, its convergence is fluctuation convergence. Graphs of f=10 and f=100 are the results of the modified algorithm. As it is shown in Figures 2 and 3, rate of convergence is increased by increasing of f and in Figure 3, fluctuation convergence of weights are disappeared.

3.2 Second method

For eliminating m's range or extent in Eq. (9), the below is proposed.

$$W_k = W^* + \left[\frac{1}{(1+2b)^{fk}}\right](W_0 - W^*) \qquad (14)$$

For achievement of this in Eq. (9), m will be considered as

$$m = \frac{1}{1+2b} \qquad (15)$$

By choosing m from Eq. (15) and substituting it in Eq. (9), the algorithm of Eq. (14) is obtained. If b and f are let to approach infinity in Eq. (14), the rate of convergence is highly increased, in which b range is a real number (b>0) and in this modified algorithm, adaptive filter do not arrive in unstable and fluctuation convergence. In Figure 4, the simulation of f=1, that it shows the increasing of b in increasing of rate of convergence.

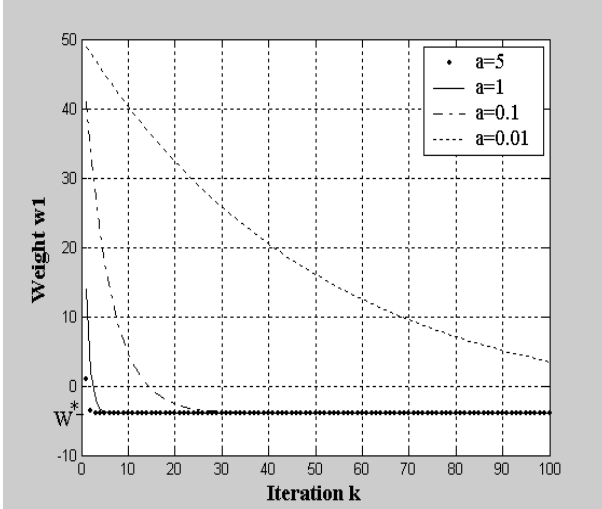


Figure 4: filter weights in accordance with different value of b.

Figure 4 shows the rate of convergence of adaptive filter weights to W^*. In Figure 4, the value of f is considered constant and by increasing the value of b, the rate of convergence will be increased.

4 The use of modified algorithms in smart antenna

Adaptive filters have extended use in smart antennas. The main use of, these are to beam forming in ideal directions and to make null in unwanted directions. By increasing the rate, these antennas algorithms can beam forming thus fast which this causes saving in energy, time and expense. Different adaptive algorithms are used for the usage of smart antennas such as LMS [1] and SER [2]. In sequel, one of the applied of smart antenna is reviewed. Figure 5 is related to the block diagram of the smart antenna [7, 10].

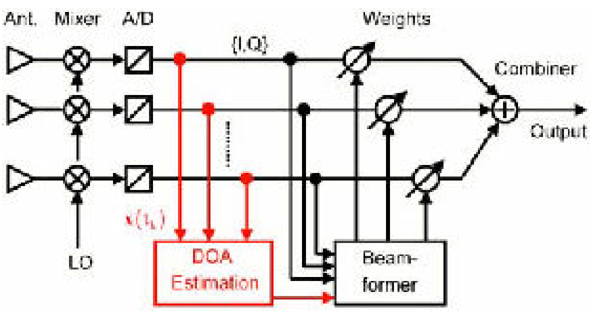


Figure 5: functional block diagram of a smart antenna system.

As Figure 5 presents, these antennas consist of three parts. First part is the antennas itself, second part is signal processing unit and third parties beam

[1] Least mean square

[2] Sequential Regression Algorithm

forming unit of antenna. The smart antennas are [7].

$$X(t) = A(\theta)S(t) + n(t) \quad (16)$$

$$A(\theta) = [a(\theta_1), a(\theta_2), ..., a(\theta_M)] \quad (17)$$

$$S(t) = [s_1(t), s_2(t), ..., s_M(t)] \quad (18)$$

where A(θ) is the steering matrix, S(t) is vector of input signal without noise and n(t) is input noise.

$$\begin{aligned} R &= E[X(t)X(t)^T] \\ &= AE[S(t)S(t)^T]A^T + E[n(t)n(t)^T] \\ &= ASA^T + \sigma^2 I \end{aligned} \quad (19)$$

where S is the source covariance matrix or input correlation matrix, $\sigma^2 I$ is the noise covariance matrix that is a reflection of the noise having a common variance σ^2 at all sensors and being uncorrelated among all sensors and I is identity matrix.

$$R = ASA^T + \sigma^2 I = Q\Lambda Q^T \quad (20)$$

$$R = Q_s \Lambda_s Q_s^T + Q_n \Lambda_n Q_n^T \quad (21)$$

where R matrix is represented according to eigenvalues where Q, Q_n and Q_s are eigenvectors and Λ is eigenvalues matrix that its main diagonal is R eigenvalues. Λ_n is eigenvalues matrix of noise that its all main diagonal entries are equal σ^2 and Λ_s is S's eigenvalues matrix. Then the out put Y_k for kth iteration is

$$Y_k = W^T X_k \quad (22)$$

By comparing the desired response d_k with the output Y_k, we produce an estimation error denoted by

$$e_k = d_k - Y_k \quad (23)$$

The LMS algorithm is

$$W_{k+1} = W_k + 2\mu e_k X_k \quad (24)$$

For the use of modified algorithm in LMS algorithm, the weight vector's convergence is checked and because of this, from members of LMS algorithm expectation is obtained [1].

$$\begin{aligned} E[W_{k+1}] &= E[W_k] + 2\mu E[e_k X_k] \\ &= E[W_k] + 2\mu E\{[d_k X_k] - E[X_k X_k^T W_k]\} \\ &= E[W_k] + 2\mu\{P - RE[W_k]\} \\ &= (I - 2\mu R)E[W_k] + 2\mu R W^* \end{aligned} \quad (25)$$

The solution of this equation is complicated by the fact that the various components of W_k in Eq. (25) is not diagonal because it contains the term 2μR, and in general R is not diagonal. The equation maybe compared with the corresponding Eq. (2) for Newton's method to distinguish between a cross-coupled and an uncoupled system. We can, however, solve Eq. (25) by transforming to the principal coordinate system. First we translate as in Eq. (30), using $V=W-W^*$, so that Eq. (25) becomes

$$V'_k = (I - 2\mu\Lambda)^k V'_0 \quad (26)$$

As Eq. (26), it is clear that LMS algorithm's convergence is like S.D.[3] algorithm, and by use of modified algorithms of S.D. method, the rate of convergence will be increased by increasing the rate of LMS algorithm and because of it the rate of antenna beam forming increases too.

5 S.D. algorithm

S.D. algorithm is [1].

$$V'_k = (I - 2\mu\Lambda)^k V'_0 \quad (27)$$

$$0 \prec \lambda_{\max}\mu \prec 1 \quad (28)$$

And V'_0 is the initial weight vector which is transferred to principal axes and V'_k is the vector of the transferred weight to principal axes.

$$V'_k = Q^T V_k \quad (29)$$

$$V_k = (W_k - W^*) \quad (30)$$

Q^T is the transpose vector of Q which is dependent on in put and V_k is weight vector of W_k which is transpose to W^* value.

6 Modified algorithm of S.D. method

With some changes in choosing μ a modified S.D. algorithm can be introduce. In S.D. method the only difference is in choice rang of μ Eq. (28). In this new method, the Eq. (27) is changed as

$$V'_k = (I - Y)^{fk} V'_0 \quad (31)$$

where f is a coefficient calculated through the appropriate value of μ. f is an integer and even number in order to prevent weights from fluctuation convergence and Y is a diagonal matrix whose main diagonal is always equal to y which will be discussed later. Rate of convergence will increase significantly with the increase of f. In Eq. (27), $(I-2\mu\Lambda)^k$ is a diagonal matrix whose main diagonal is equal to $(1-2\mu\lambda_n)^k$ and in the proposed algorithm of Eq. (31), $(I-Y)^{fk}$ is a diagonal matrix whose main diagonal is $(1-y)^{fk}$. Considering what discussed above and setting $(I-2\mu\Lambda)^k$ equal to $(I-Y)^{fk}$ the S.D. algorithm with modified algorithm. For making Eq. (31), the steps are processed in below from in which it is first considered.

$$(1 - 2\mu\lambda_n) = (1 - y)^f \quad (32)$$

[3] Steepest Descent

In Eq. (32), λ_n are the eigenvalues of matrix R. According to $(1\text{-}y)^f$ in Eq. (11) and its comparison with Eq. (32), we have

$$Z_n = 0.5\lambda_n \sum_{h=1}^{h=f} \frac{(-1)^{h+1} f!}{h!(f-h)!} y^h \qquad (33)$$

In Eq. (33), if y and f are constant and λ_n is varying then Z_n will have different values, therefore μ is defined as a Z_n vector. If we substitute Z_n for μ in Eq. (27) we will have

$$V'_k = (I - 2Z_n\Lambda)^k V'_0 \qquad (34)$$

In order to change $(I\text{-}2Z_n\Lambda)$ in proper form, that is, diagonal matrix forms, another matrix is define as

$$M = Z_n I \qquad (35)$$

And Z_n is replaced whit M in Eq. (34) will lead to (36).

$$V'_k = (I - 2M\Lambda)^k V'_0 \qquad (36)$$

$(I\text{-}2M\Lambda)^k$ is a diagonal matrix whose main diagonal's elements are equal to $(1\text{-}2Z_n\lambda_n)^k$. Since in Eq. (33), Z_n depended on λ_n in way that y and f have a constant value, $Z_n\lambda_n$ will always have constant value. By substituting Z_n form Eq. (33) in Eq (35) and Eq. (34) the proposed algorithm $V'_k=(I\text{-}Y)^{fk}V'_0$ as in Eq. (31) will be gained. In Eq. (31), $(I\text{-}Y)^{fk}$ is a diagonal matrix whose main diagonal is always the constant value of $(1\text{-}y)^{fk}$. Thus the modified algorithm can be shown as

$$V'_k = (1-y)^{fk} V'_0 \qquad (37)$$

Eq. (37) is the modified algorithm of S.D. method. In this method the weights unlike S.D. algorithm are converged with the regular same time constant and the rate of convergence is increased. In order to simulate and compare the modified algorithm with S.D. algorithm, μ and y are selected in a way that the value of $(1\text{-}2\mu\lambda_{max})$ in S.D. algorithm is equal to the value of (1-y) in modified algorithm, thus that the effect of f in increasing rate of convergence be observable. In the simulate example, the values $\lambda_1=\lambda_{max}=1.5$ and $\lambda_2=0.5$ are calculated, thus, Figure 6, $\mu=0.0066$ for S.D. algorithm and y=0.02 in the modified algorithm are considered.

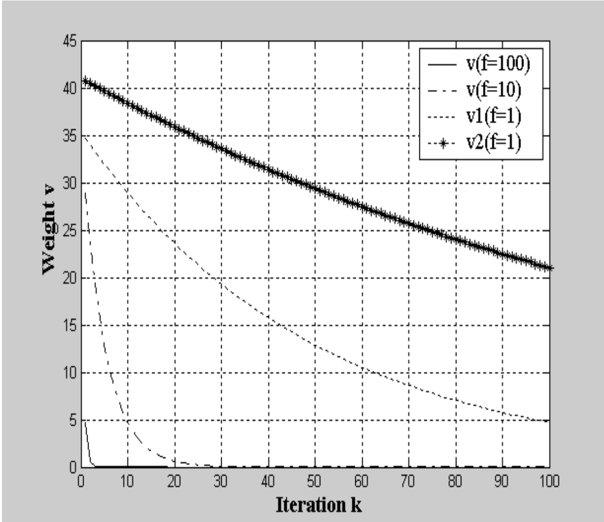


Figure 6: Filter weights in accordance with different value of f.

Figure 6 shows the rate of convergence of adaptive filter weights to W^*. Since in S.D. algorithm the weights W_k, is transfered to principal axes, therefore all the converged weight on approach zero. Graphs f=1 is related the result of convergence of the two weights of S.D. algorithm's and graphs f=10 and f=100 are the results of the modified S.D. algorithm. In both states f=10 and f=100 the modified S.D. algorithm is converging with the same time constant. The increase in rate of convergence and the omission of convergence with different time constant are shown in Figure 6. Increase in rate of convergence and determination of f are at you will. Without increase in noise fluctuation this can be done. In the next section variations of the value of estimated noise with the application of the discussed methods will be studied.

7 The review of gradient estimation noise in modified algorithms

7.1 Gradient estimation noise in Newton's algorithm

It is examined the effect of this noisy gradient estimate on the weight vector solution, with Newton's method [1].

$$W_{k+1} = W_k - \mu R^{-1}\hat{\nabla}_k \qquad (38)$$

$$\hat{\nabla}_k = \nabla_k + N_k \qquad (39)$$

$$\nabla_k = 2RV_k \qquad (40)$$

∇_k is the true gradient, $\hat{\nabla}_k$ is the gradient estimated and N_k is the gradient estimation noise that each of these are in kth iteration. By extension the equations and substituting on Newton's algorithm, the Eq. (41) is obtained.

$$V'_{k+1} = (I - 2\mu)V'_k - \mu\Lambda^{-1}N'_k \quad (41)$$

$$N'_k = Q^{-1}N_k \quad (42)$$

By extension the Eq. (41), V'_k is fund as

$$V'_k = (I-2\mu)^k V'_0 - \mu\Lambda^{-1}\sum_{n=0}^{n=k-1}(1-2\mu)^n N'_{k-n-1} \quad (43)$$

Thus we have a solution as in Eq. (2) for Newton's method, expect that here we are in the principal-coordinate system and the gradient noise, N'_k, is included at each step. In Eq. (2) we could let k approach infinity and obtain the optimum solution, $W=W^*$, or $V'=0$, but here there is a residual error due to the gradient noise. If k is let to approach infinity in Eq. (43) and assume that μ is in the stable range as in Eq. (8), the factor $(1-2\mu)^k$ will become negligible and the following "steady-state" solution is obtained.

$$k \to \infty \Rightarrow (1-2\mu)^k \to 0 \quad (44)$$

$$V'_k = -\mu\Lambda^{-1}\sum_{n=0}^{\infty}(1-2\mu)^n N'_{k-n-1} \quad (45)$$

This gives us the steady-state error in the weight vector for Newton's method in terms of the input eigenvalues in Λ^{-1} and the gradient noise in the series of values given by Eq. (42). The remained term is because of noise that if there is noise, its amount is equal Eq. (45). By choosing μ from Eq. (12) and substituting it in Eq. (43), the Eq. (46) is obtained.

$$V'_k = (I-2m)^{fk} V'_0 - \mu\Lambda^{-1}\sum_{n=0}^{n=k-1}(1-2m)^{fn} N'_{k-n-1} \quad (46)$$

$$V'_k = -\mu\Lambda^{-1}\sum_{n=0}^{\infty}(1-2m)^{fn} N'_{k-n-1} \quad (47)$$

By comparing Eq. (47) with Eq. (45), noise fluctuation is decreased by compatible increasing of f and by using the second method in Eq. (47), V'_k is fund as

$$V'_k = -\mu\Lambda^{-1}\sum_{n=0}^{\infty}\frac{1}{(1-2m)^{fn}} N'_{k-n-1} \quad (48)$$

It is shown that in given methods, increasing of rate of convergence is done with decreasing of noise fluctuation.

7.2 Gradient estimation noise in S.D. algorithm

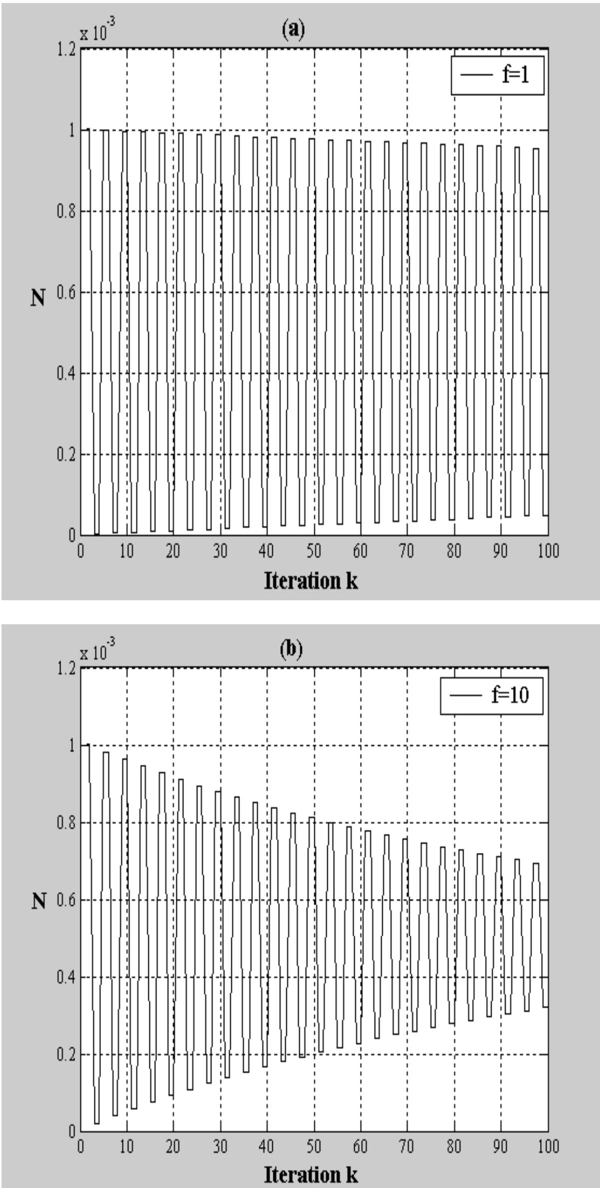
It is examined the effect of this noisy gradient estimate on the weight vector solution, with S.D. method [1].

$$V'_k = -\mu\sum_{n=0}^{\infty}(1-2\mu\Lambda)^n N'_{k-n-1} \quad (49)$$

By choosing μ from Eq. (31) till (37) and substituting it in Eq. (49), the gradient estimation noise in modified S.D. algorithm of Eq. (50) is obtained.

$$V'_k = -\mu\sum_{n=0}^{\infty}(1-y)^{fn} N'_{k-n-1} \quad (50)$$

By comparing the Eq. (49) with Eq. (50), noise fluctuation is decreased by compatible increasing of f which it represents increasing of the rate of convergence. By considering gradient of estimation noise -1<N<1 in upper given example, in Figure 7 the results are shown.



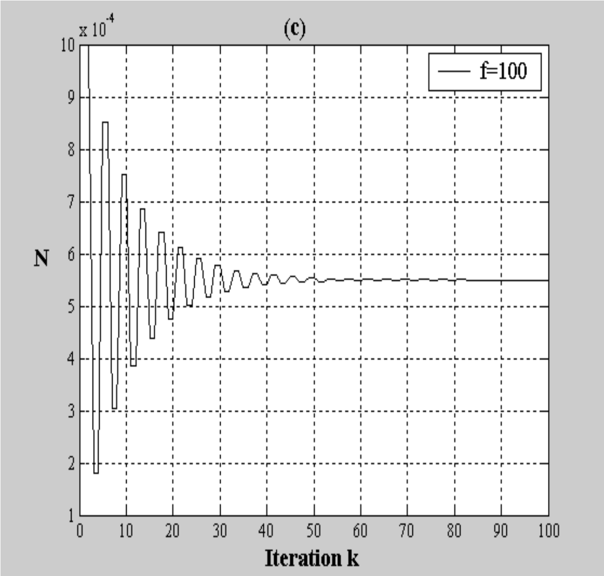


Figure 7: The gradient estimation noise in accordance with different value of f.

Figure 7 shows the gradient estimation noise in accordance with different value of f for μ=0.001 and m=0.001. Figure 7-a in which f=1 is related to the result of Newton's algorithm and Figures 7-b and 7-c in which f=10 and f=100 are show the results of the new Newton's algorithm. These simulated results show the decreasing way of gradient estimation noise and its fluctuation by increasing f for modified algorithm.

8 Conclusion

In this paper two methods proposed for increasing of rate of convergence. In first method the rate of convergence is being increased according determination of f. f is a factor get by suitable selection of μ in modified algorithm. Concerning the results in simulation and mathematical relationship, the rate of convergence will considerably increase by increasing of f. In second method by choosing proper b and provided for the selection of b range is a real number (b>0) and limitation for stable range of μ or m is omitted. In this method, the fluctuation convergence and unstability of filter is prevented and there is no need for f to be even. In the given method for S.D. in which μ is defined as vector, causes of making an active algorithm for evaluation of weight vector. In this method the weights unlike primary algorithm are converged with the regular same time constant and in contrast with Newton's method in its primary algorithm have no need to inverse input matrix of R^{-1}. According to the simulated example for estimation noise of algorithms, it is noted that by increasing f not only the speed increases but also noise fluctuation decreases partly and the filter weights will converge to the optimum value W^*.

Acknowledgements

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