Tunable Constant Q Band-Pass Filter Design Using q and k Values

This paper describes a method for designing a tunable, coupled-resonator band-pass filter using q and k values. The method results in a filter that has a constant Q and a consistent filter characteristic as it is tuned from low to high frequency. The q and k values are either calculated or are taken from published tables. A symmetric design based on a N=2 low-pass prototype and using lossless components is assumed for this design example.

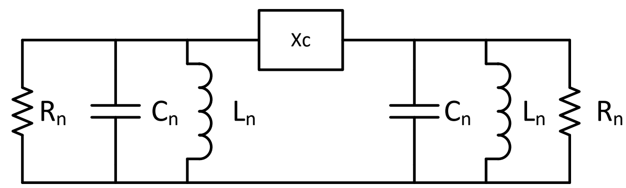


Figure 1. Prototype filter.

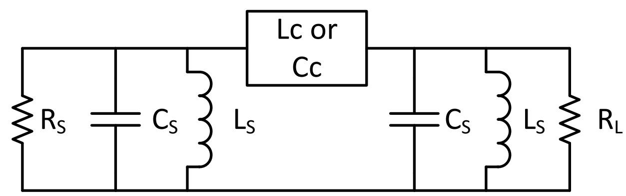


Figure 2. Realized filter with inductive or capacitive resonator coupling.

R_N = Nodal resistance
C_N = Nodal capacitance
L_N = Nodal inductance
X_C = Coupling reactance, either inductive (L_C) or capacitive (C_C).
R_S = R_L = source and load resistance
C_S = shunt capacitance
L_S = shunt inductance
C_C = coupling capacitance
L_C = coupling inductance

Given q_N & k_N:

Filter Q, $Q_{bp} = \frac{\omega_0}{\Delta\omega_{-3dB}}$ (1)

$$K_N = \frac{k_N}{Q_{bp}} \quad (2)$$

$$Q_N = q_N Q_{bp} \quad (3)$$

$$R_N = \omega_0 Q_N L_N \quad (4)$$

$$\omega_0 = \frac{1}{\sqrt{L_N C_N}} \quad (5)$$

Coupling structure and shunt elements:

$$C_C = C_N K_N = \frac{C_N k_N}{Q_{bp}} \quad (6) \qquad C_S = C_N - C_C \quad (7)$$

$$L_C = \frac{L_N}{K_N} = \frac{L_N Q_{bp}}{k_N} \quad (8) \qquad L_S = \frac{1}{\frac{1}{L_N} - \frac{1}{L_C}} \quad (9)$$

To make the filter tunable, either the nodal C or nodal L (or both) must change. From (6) it is seen that with capacitive coupling, C_C varies with nodal capacitance C_N. From (8), the inductive coupling element, L_C varies with nodal inductance L_N.

Conclusion:

> If it is desired to hold the coupling element constant while tuning a filter by varying C_N, then inductive coupling would be the type to use. Conversely, if the filter is tuned with variable inductance then capacitive coupling should be used.

Notice that these are true only for a filter whose Q, Q_{BP} is constant as the filter center frequency is tuned.

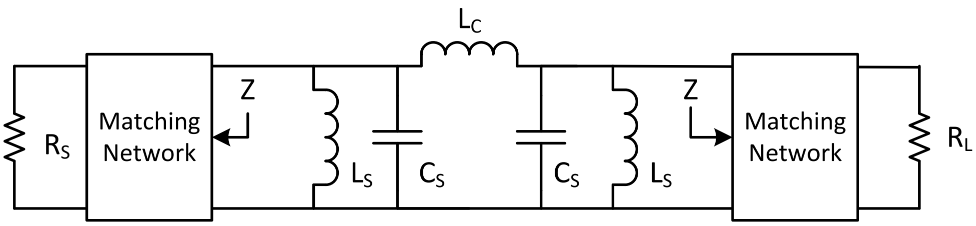
But from (3) & (4): $$Q_{bp} = \frac{R_N}{\omega_0 L_N q_N} \quad (10)$$

$$\therefore \quad Q_{bp} \propto 1/\omega_0$$

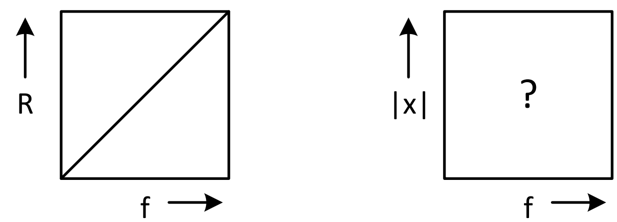
In order to have the Q remain constant as the filter is tuned, then R_N the numerator of (10) needs to vary linearly with frequency as the filter is tuned from ω_{LO} to ω_{HI}. This will give a filter with a non-changing transfer function and constant Q_{bp}.

From (4), $\frac{R_N}{\omega_0} = Q_{bp} L_N q_N$. With Q_{bp} held constant as ω_0 is increased, R_N must increase proportionally.

We want a matching network that converts a fixed resistance on one side to one that varies proportionally with frequency on the other.

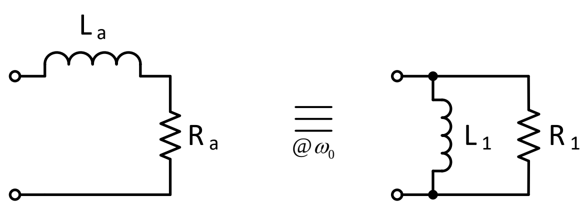


Desired R of the matching network presented to the resonant nodes:



Any reactive component of the matching network will be absorbed by either the Ln or Cn.

Consider this narrow band equivalency:



$R_1 = R_a + \frac{\omega_0^2 L_a^2}{R_a}$ (see Appendix)

Using this network as the matching circuit changes Figure 2 as follows:

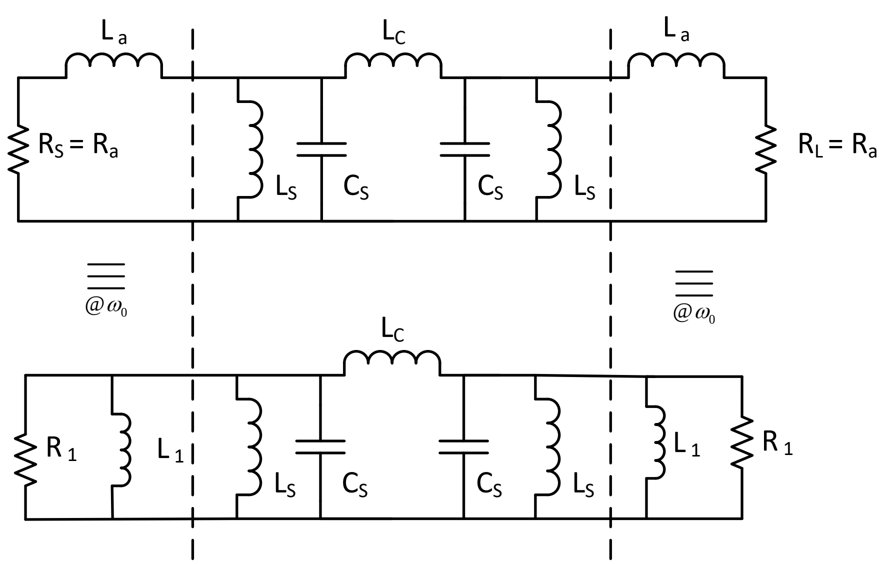


Figure 3. Filter with matching network and narrowband equivalent.

R1 will be the nodal resistance, R_N and will increase somewhat non-linearly with frequency but can be made to match the desired value of R1 at the two frequency tuning extremes, ω_{LO} and ω_{HI}. Between the two frequency extremes, R1 will have a positive error relative to the ideal value of RN. A simulation of the network will show whether it has an unacceptable effect on the filter's transfer function. It can be mitigated somewhat by reducing the frequency range for the calculation of the network. This will split the error between those frequencies above and below the calculated range, which will be negative, and the those between the calculated range, which will be positive.

$$R_{1\_LO} = R_a + \frac{\omega_{LO}^2 L_a^2}{R_a} \quad (11) \qquad R_{1\_HI} = R_a + \frac{\omega_{HI}^2 L_a^2}{R_a} \quad (12)$$

Since R_1 increases proportionally with frequency:
$R_{1\_HI} = mR_{1\_LO}$, where $m = \frac{\omega_{HI}}{\omega_{LO}}$, (13)

From (11), $mR_{1\_LO} = mR_a + \frac{m\omega_{LO}^2 L_a^2}{R_a}$
From (12) and (13), $mR_{1\_LO} = R_a + \frac{m^2\omega_{LO}^2 L_a^2}{R_a}$

Equating:

$$R_a + \frac{m^2\omega_{LO}^2 L_a^2}{R_a} = mR_a + \frac{m\omega_{LO}^2 L_a^2}{R_a} \xrightarrow{yields} R_a^2 = m\omega_{LO}^2 L_a^2$$

$$L_a = \frac{1}{\sqrt{m}}\frac{R_a}{\omega_{LO}} \quad (14)$$

$$R_a = R_{1\_LO}\left(\frac{m}{m+1}\right) \quad (15)$$

$$L_1 = L_a + \frac{R_a^2}{\omega_0^2 L_a} \quad (16)$$

$$L_S = \frac{1}{\frac{1}{L_N} - \frac{1}{L_1} - \frac{1}{L_C}} \quad (17)$$

L1 will vary some with frequency but it is normally such a large value that its effect on Ls is minimal. To get realizable values the value of Ra will usually be increased. This can be matched to 50 ohm using a transformer. Normally one would design the filter to meet attenuation and passband requirements at the two tuning extremes, then using a spreadsheet loaded with appropriate equations, adjust the nodal inductance and perhaps filter Q to get an Ra that can be matched by a convenient transformer ratio and realizable component values.

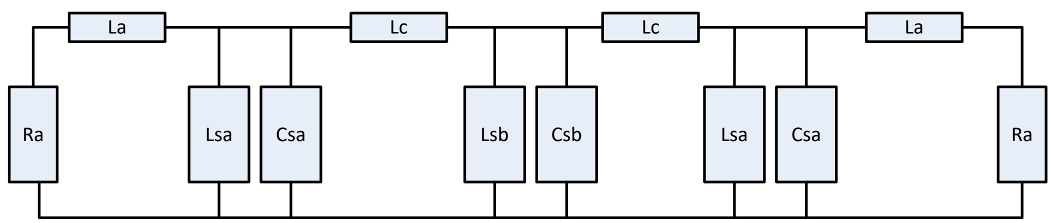
The design method shown for non-dissipative components can be extended to dissipative components and higher numbers of sections by using the corresponding q and k values associated with those components and sections.

Example Design: Tunable Preselector

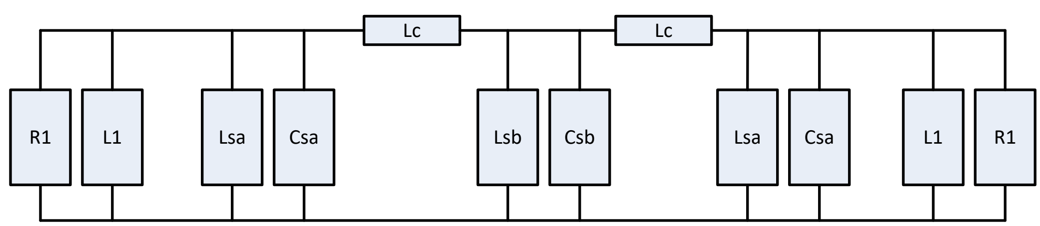
Frequency tuning range: 118 – 152 MHz
When tuned to 137 MHz we want the attenuation at 157 MHz to be at least 35 dB. According to Zverev, a N=3 Butterworth has a 3 dB to 35 dB bandwidth ratio of about 3.8.

When tuned to 137 MHz, the bandwidth at the 35 dB point is $157 - \frac{137^2}{157} \cong 37.5\ MHz$. The bandwidth at the 3 dB point is $\frac{37.5}{3.8} = 8.9\ MHz$. The filter Q is then $\frac{137}{9.9} = 13.8$.

The filter:



$$\equiv @\omega_0$$

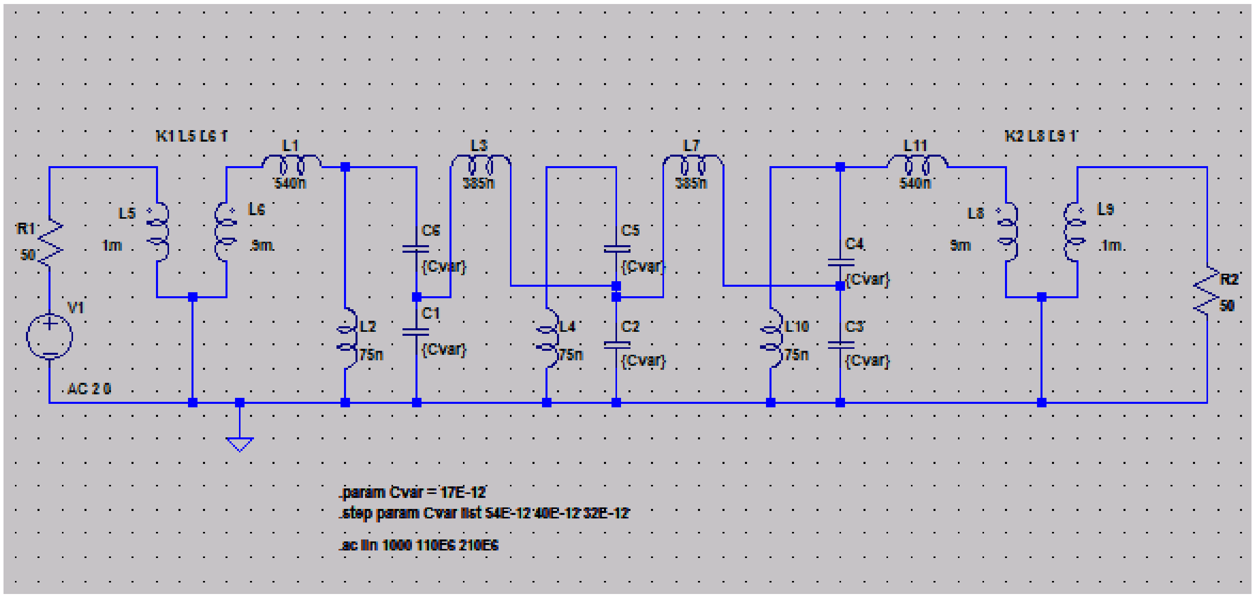


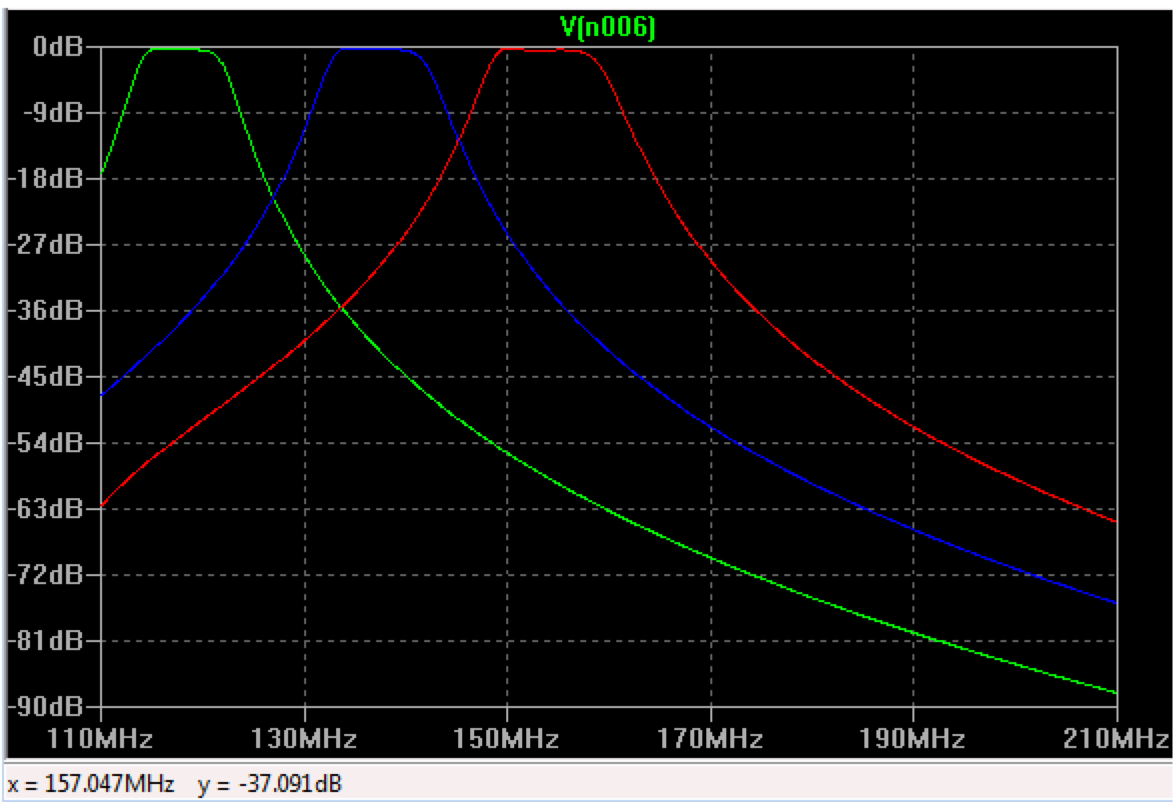
Flo = 118 MHz
Fhi = 152 MHz
Filter, n=3 Butterworth
$q_1 = q_N = 1.0$
$k_{12} = k_{34} = 0.7071$

L_N and Q were manipulated to give realizable values and to give an R_A value that was a transformable value from 50 ohms.

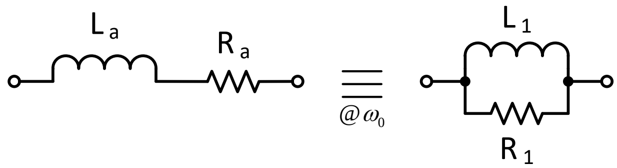
Q = 16.0
L_N = 68 nH
$L_{sa} = L_{sb}$ = 75 nH
L_a = 540 nH
R_a = 450 ohm (50 ohm with 9:1 transformer)
L_C = 1539 nH (filter topology will reduce this to $L_C/4$)
$C_{sa} = C_{sb}$
C_{sa_118MHz} = 27 pF
C_{sa_137MHz} = 20 pF
C_{sa_152MHz} = 16 pF

This final filter and its response is shown below:


K1 L5 L6 1
L1
540n
L3
385n
L7
385n
L11
540n
K2 L8 L9 1
R1
50
L5
1m
L6
.9m
C6
{Cvar}
C1
{Cvar}
L2
75n
L4
75n
C5
{Cvar}
C2
{Cvar}
C4
{Cvar}
L10
75n
C3
{Cvar}
L8
.9m
L9
.1m
R2
50
V1
AC 2 0
.param Cvar = 17E-12
.step param Cvar list 54E-12 40E-12 32E-12
.ac lin 1000 110E6 210E6


V(n006)
0dB
-9dB
-18dB
-27dB
-36dB
-45dB
-54dB
-63dB
-72dB
-81dB
-90dB
110MHz
130MHz
150MHz
170MHz
190MHz
210MHz
x = 157.047MHz y = -37.091dB

Appendix - Narrow-Band Approximations



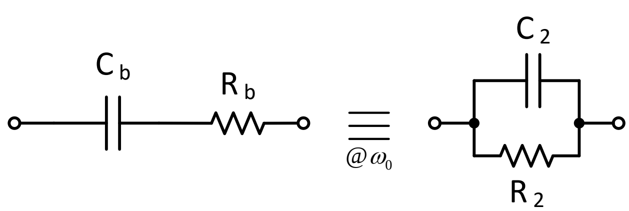
$$L_1 = L_a + \frac{R_a^2}{\omega_0^2 L_a} \quad \text{(A-1)}$$

$$R_1 = R_a + \frac{\omega_0^2 L_a^2}{R_a} \quad \text{(A-2)}$$

$$L_a = \frac{L_1 R_1^2}{R_1^2 + \omega_0^2 L_1^2} \quad \text{(A-3)}$$

$$R_a = \frac{\omega_0^2 L_1^2 R_1}{R_1^2 + \omega_0^2 L_1^2} \quad \text{(A-4)}$$

$$L_a = \sqrt{\frac{R_a(R_1 - R_a)}{\omega_0^2}} \quad \text{(A-5)}$$



$$C_2 = \frac{C_b}{1 + \omega_0^2 C_b^2 R_b^2} \quad \text{(A-6)}$$

$$R_2 = R_b + \frac{1}{\omega_0^2 C_b^2 R_b} \quad \text{(A-7)}$$

$$C_b = C_2 + \frac{1}{\omega_0^2 R_2^2 C_2} \quad \text{(A-8)}$$

$$R_b = \frac{R_2}{1 + \omega_0^2 C_2^2 R_2^2} \quad \text{(A-9)}$$

$$C_b = \sqrt{\omega_0^2 R_b (R_2 - R_b)} \quad \text{(A-10)}$$

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